Capacity Benefits of Antenna Coupling

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Abstract—Antennas can couple if they are placed near each other, such as on portable wireless devices. Such coupling is often viewed as a nuisance that is a detriment to system performance. We show how coupling, if exploited properly, can lead to higher achievable data rates than an equivalent uncoupled system. Our conclusions build on some recent results on the bandwidth of coupled radiofrequency systems.

I. INTRODUCTION

The use of multi-antenna systems for single user communications was popularized by the analyses of [1] and [2] which show that superior rates are available to MIMO systems compared to their SISO counterparts. These analyses generally focus on narrowband systems with idealized antennas.

We use a wideband analysis to show that antenna coupling can lead to very high capacity in a multi-antenna system. The capacity of a wideband coupled system is formulated in (11), with a broadband antenna matching bound constraint (13). We use two coupled dipoles to demonstrate that a 35% increase in the capacity is possible over using a pair of uncoupled dipoles.

We do not idealize the antennas or the radio-frequency (RF) matching network used to drive the antennas. Realistic radio-frequency simulations are used to model the antennas, their coupling, and far-field patterns.

We exploit the fact that the bandwidth of an RF system is often limited by the ability to match a wideband amplifier with a (relatively) narrowband antenna. Under some conditions, the ability to accomplish this match improves in the presence of antenna coupling. With increased bandwidth, the system capacity can also increase.

In RF systems, impedance mismatch between amplifiers (sources) and antennas (loads) causes power to be reflected from the antennas back to the amplifiers. Matching circuits are used in between to minimize the reflected power. Single source single load matching is well understood due to the theory developed by Bode [3] and Fano [4]. The theory states that perfect matching is not possible over a continuous range of frequencies. The bandwidth of the system is defined as the set of frequencies over which the power reflection is below a threshold.

For multiple sources and loads, a recent theory presented in [5] proposes a generalization of [3] and [4]. The notions defined therein, such as reflection and bandwidth, provide a means to analyze coupled multiple antennas connected to amplifiers through matching circuits. In this paper, we draw from the results in [5], to analyze the effect coupling has on the capacity of a system with coupled antennas at the transmitter.

When driving closely spaced antennas, energy can couple from one antenna to another. This coupling can also lead to correlation between the signals either transmitted or received from the antennas. Some works, such as [6], show that correlation can be used to the advantage of capacity in MIMO systems. Others analyze the capacity of systems with coupled circular antenna arrays [7], [8]. However, as shown in [9], even coupled antennas that are properly matched can have zero correlation. We exploit this effect in our analysis.

We focus on antennas used in a transmit configuration, but the analysis can also be used to accommodate antennas at a receiver. We use two identical vertically-oriented half-wavelength dipole antennas, placed \( d \) apart and connected through a matching network to two amplifiers. The dipoles are designed for a center frequency of 2.4 GHz, making the wavelength \( \lambda = 125 \) mm. The dipoles are used over a bandwidth of 4 GHz. It is shown that the dipoles can be driven to achieve higher bandwidths, and hence higher capacity, without introducing channel correlation. We treat only one end of the communication link, and hence ignore the receiver configuration; the receiver is assumed to have a single antenna with unlimited bandwidth.

Figure 1 shows the capacity of the system with coupled dipoles as a function of separation. The baseline for comparison is the capacity of a system with an uncoupled set of the same dipoles. The maximum capacity, which is 35% higher than the uncoupled dipoles, occurs at \( d = 0.23\lambda \). The transmit power is fixed to produce 10 dB SNR at the receiver with uncoupled dipoles.

The rest of this paper derives the results needed for Figure 1. Section II introduces the setup of the considered communication system in more detail, along with an overview of the relationship between coupling and correlation. Section III presents the analysis of the capacity of the MIMO system, starting with (11) leading to (26). Finally, Section IV describes the simulation setup resulting in the capacity plot of Figure 1. Section IV also briefly discusses a source of frequency selectivity in the channel that is not due to multipaths of the antenna signal, by examining the the far-field patterns of the antennas in this wideband setup.
Fig. 1: MIMO capacity (26) of the system with coupled dipoles at the transmitter as a function of distance $d$ expressed as fractions of the wavelength $\lambda$. The dashed line is the capacity of the same system setup with uncoupled (isolated) dipoles with an SNR of 10 dB at the receiver.

II. SETUP AND PROBLEM FORMULATION

In this section, we describe the transmitter configuration and show the relationship between correlation and coupling.

A. Communication System Setup

We focus on a two-transmit one-receive antenna system, where channel state information is not available to the transmitter. Figure 2a captures the transmitter architecture, where there are two identical lossless dipoles, placed a distance $d$ apart, with the geometry shown in Figure 2b. Our model for the dipoles is realistic; they are drawn in an RF simulation tool (HFSS), and allowed to interact (couple) as determined by Maxwell’s equations. They are $\lambda/2$ long, where $\lambda = 125$ mm, and they are assumed to be in free space. No idealizations are made about their near-field interactions or far-field patterns.

Because the antennas are realistic, they present themselves with impedance that varies with frequency. The amplifiers driving the antennas have characteristic impedance $Z_0$, and must be matched to the antennas using a matching network. The matching network is a key component in determining the bandwidth of the overall system, since a poor match at the transmitter as a function of distance $d$ at the receiver.

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The two-input two-output matching network is lossless and therefore has a unitary scattering matrix. We use the network structure shown in the first dotted box in Figure 2a, where two separate two-port matching networks are connected to the loads through a 180°-hybrid. That such an architecture can be used without loss of generality is shown, for example, in [10]. The blocks labeled “Matching Network” are unspecified. We specify them only indirectly by bounding their matching performance, as shown later. We denote $S_{LM}(d, \omega)$ to represent the scattering matrix seen by the sources that are attached to the inputs of the matching network.

The receiver, with a single antenna, is idealized to have an isotropic radiation pattern with perfect matching across the entire bandwidth utilized by the transmitter. The scattering environment is assumed rich, and the fading is represented by a vector

$$H(\omega) = [h_1(\omega) \ h_2(\omega)].$$

The entries $h_1(\omega)$ and $h_2(\omega)$ are zero-mean, unit-variance complex Gaussian random variables which are potentially correlated, with correlation coefficient

$$\rho(d, \omega) = E[h_1(\omega)h_2^*(\omega)].$$

Such correlation could occur if the antenna patterns overlap to illuminate the same scatterers. We have more to say about this correlation and the dependence of $H(\omega)$ on frequency later.

The received signal is expressed by

$$y(d, \omega) = H(\omega)s(d, \omega) + n$$

where $s$ is the 2-by-1 transmitted signal vector, and $n$ is an additive scalar complex Gaussian noise with variance $N_0$.
We obtain an expression for $\mathbf{s}(d, \omega)$, the signals that are radiated, in terms of $\mathbf{a}(\omega)$, the signals incident on the matching network. We use a conservation of energy argument. The radiated power is denoted by $P_T(d, \omega)$. The power delivered to the matching network is denoted by $P_D(d, \omega)$. Since the matching network and the antennas are lossless, we have

$$P_T(d, \omega) = P_D(d, \omega)$$

(5)

$$s^\dagger(d, \omega)\mathbf{s}(d, \omega) = \|\mathbf{a}(\omega)\|^2 - \|\mathbf{E}(\omega)\|^2$$

(6)

$$s^\dagger(d, \omega)\mathbf{s}(d, \omega) = \mathbf{a}^\dagger(\omega)(I - S_{LM}^\dagger(d, \omega)S_{LM}(d, \omega))\mathbf{a}(\omega)$$

(7)

for any $\mathbf{a}(\omega)$. Consequently, the transmitted signal can be expressed by

$$\mathbf{s}(d, \omega) = T(d, \omega)\mathbf{a}(\omega)$$

(8)

where

$$T^\dagger(d, \omega)T(d, \omega) = I - S_{LM}^\dagger(d, \omega)S_{LM}(d, \omega)$$

(9)

Hence, the covariance matrix of $\mathbf{s}(d, \omega)$ is

$$K(d, \omega) = T(d, \omega)Q(\omega)T^\dagger(d, \omega)$$

(10)

where $\mathbb{E}[\mathbf{a}(\omega)\mathbf{a}^\dagger(\omega)] = Q(\omega)$.

The capacity of the system, in bits/second, can then be defined as a function of distance, by integrating over the frequencies where the RF signal resides

$$C(d) = \max_K \int_0^\infty \mathbb{E} \left[ \log |I + \frac{1}{N_0}H(\omega)K(d, \omega)H^\dagger(\omega)| \right] d\omega$$

subject to

$$\int_0^\infty tr(Q(\omega)) d\omega \leq P$$

(12)

and

$$\int_0^\infty \log \frac{1}{r(d, \omega)} d\omega \leq b(d)$$

(13)

where $|.|$ is the determinant, $tr(.)$ is the trace, and the power reflection ratio $r^2(d, \omega)$ [5] is defined as

$$r^2(d, \omega) = \frac{1}{2}\|S_{LM}(d, \omega)\|^2_F$$

(14)

with $\|\|_F$ denoting the Frobenius norm of a matrix.

The constraint in (12) is the usual trace power constraint on the inputs. The constraint in (13) is derived in [5], and applies to any matching network. Note that $0 \leq r(d, \omega) \leq 1$, where small values of $r$ indicate that little power is reflected, and values close to one indicate that most power is reflected and therefore not delivered to the antennas. If we set a threshold $\tau > 0$ and consider the range of frequencies (bandwidth) for which

$$r(d, \omega) < \tau,$$

(15)

then (13) bounds this bandwidth as a function of $\tau$.

In general, the right-hand side of (13) is non-trivial to compute and involves modeling the antennas using a numerical Maxwell equation solver (such as HFSS or CST), and fitting the resulting model to a rational function whose poles and zeros are computed. We omit the details here, and suffice to say that $b(d)$ can be computed using the recipe outlined in [5].

Because the RF properties of the antennas vary with $d$, $b(d)$ is a function of $d$, and therefore the ability to match the antennas also varies with $d$. Hence, the capacity, as computed in (11) is a function of spacing. Since (13) is a bound that applies to all possible passive matching networks, we do not necessarily know if the bound is achievable or the exact two-port networks needed in Figure 2a. Nevertheless, we treat the bound as if it is achievable through the remainder of the discussion.

B. Correlation and Coupling

To compute the expectation in (11), we need to account for any correlation in the entries of $H(\omega)$ that may result from the antenna coupling.

Let

$$S_{LM}(d, \omega) = \begin{bmatrix} s_{11}(d, \omega) & s_{12}(d, \omega) \\ s_{21}(d, \omega) & s_{22}(d, \omega) \end{bmatrix}.$$  

(11)

We use a result in [9] where it is shown that the envelope correlation between the components of a signal transmitted through a matched load comprising two antennas having an overall scattering matrix $S_{LM}(d, \omega)$, is

$$|\rho(d, \omega)|^2 = \frac{|s_{11}^* s_{12} + s_{21}^* s_{22}|^2}{(1 - |s_{11}|^2 + |s_{21}|^2)(1 - |s_{22}|^2 + |s_{12}|^2)}$$

(16)

where the $(d, \omega)$ dependence is not explicitly shown on the right-hand side.

Note that $\rho(d, \omega)$, defined in (3), is needed to compute the expectation in (11). The result in (16) has two important consequences. First, if the scattering matrix $S_{LM}(d, \omega)$ is diagonal, the correlation is zero. In other words, even though the antennas are coupled, the matching network can potentially be designed so that the correlation is zero. This decoupling effect is possible through a decoupling network [11]. Second, by maintaining a small value of $r(\omega)$ as defined in (14), we can make a correlation value that is as small as desired. For example, by enforcing (15), we make $\rho(d, \omega) \approx r^2$.

III. BANDWIDTH AND CAPACITY

We now explicitly compute (11) for our two-dipole example.

A. Bandwidth

The column vectors in (2) correspond to excitations of the even (in-phase) and odd (out-of-phase) modes of the antennas. It is shown in [12] that for circulant loads, a bound of the form in (13), is the arithmetic mean of the bounds of the even and odd modes given by

$$\int_0^\infty \ln \frac{1}{r_e(d, \omega)} d\omega \leq b_e(d)$$

(17)

$$\int_0^\infty \ln \frac{1}{r_o(d, \omega)} d\omega \leq b_o(d)$$

(18)

where

$$\Lambda_{LM}(d, \omega) = \begin{bmatrix} r_e(d, \omega) & 0 \\ 0 & r_o(d, \omega) \end{bmatrix}$$

(19)
is the matrix of eigenvalues of \( S_{LM} \). The bounds in (17) and (18) capture all the information in (13). We omit explicit derivation and computation of \( b_e(d) \) and \( b_o(d) \), and display them in Figure 3.

Furthermore, (17) and (18) can be converted to bandwidth bounds by setting a threshold \( \tau \) such that

\[
    r_e(d, \omega) = r_o(d, \omega) = \tau. \tag{20}
\]

Then

\[
W_e(d) \leq \frac{b_e(d)}{\ln \frac{1}{\tau}}, \quad W_o(d) \leq \frac{b_o(d)}{\ln \frac{1}{\tau}} \tag{21}
\]

Therefore, the integral constraint in (13) can be replaced by the bandwidth constraints in (21). Generally, \( W_e > W_o \) for the two dipoles; this is also reported in [8]. Hence, part of the spectrum will have the two modes present, and we call that the overlapping region. The other part which we call the non-overlapping region, contains only the wider of the modes. The overlapping and non-overlapping regions are defined as

\[
W_{ov}(d) = \min\{W_e(d), W_o(d)\} \tag{22}
\]

\[
W_{non}(d) = \max\{W_e(d), W_o(d)\} - W_{ov}(d) \tag{23}
\]

The bands in (22) and (23) are used to simplify the capacity in (11).

The expression for \( K(d, \omega) \) in (10) is dependent on \( S_{LM}(d, \omega) \) which is diagonal for the setup in Figure 2a. Therefore, by (21)–(23) and using \( W_e > W_o \), we find

\[
S_{LM}(d) = \Lambda_{LM}(d) = \begin{cases} 
    \Lambda_{ov}(d) & \omega \in W_{ov}(d) \\
    \Lambda_{non}(d) & \omega \in W_{non}(d)
\end{cases}
\tag{24}
\]

where the dependence on \( \omega \) is no longer needed. By (24), \( T(d) \) is diagonal, and (9) can be rearranged into

\[
K(d, \omega) = Q(\omega) \left( I - S_{LM}^\dagger(d)S_{LM}(d) \right). \tag{25}
\]

Using (24) and (25) in (11), we find that the integral splits in two over the overlapping and non-overlapping regions, with the components in each being constant in frequency. It can be shown using [1] that the optimal \( Q(\omega) \) is a multiple of identity. Setting the threshold to a constant small value serves to limit the operating frequencies of the system to regimes where the reflection is low. Because \( S_{LM}(d) \) is diagonal, \( \rho(d, \omega) \) in (16) is zero, making the fading coefficients uncorrelated.

**B. Capacity**

Using (21)–(25), we may evaluate (11) as

\[
C(d) = \max_{P_{ov}} \left\{ W_{ov}(d)E\left[ \log \left( 1 + \frac{P_{ov}(d)}{2N_0W_{ov}(d)} (1 - \tau^2)HH^\dagger \right) \right] \right\} + W_{non}(d)E\left[ \log \left( 1 + \frac{P - P_{ov}(d)}{N_0W_{non}(d)} (1 - \tau^2)|h_1|^2 \right) \right] \tag{26}
\]

where \( P_{ov} \) is the power allocated in \( W_{ov} \), with \( P, \tau, W_{ov} \), and \( W_{non} \), defined in (12), (20), (22), (23) respectively.

To complete the analysis, we examine the capacity under low and high SNR regimes.

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**IV. Simulation and Results**

This section provides the details of the simulation leading to the results of Figure 1. The transmitters are two parallel half-wavelength dipoles at 2.4 GHz spaced \( d \) from one another. Since the bound values in (21) depend entirely on the scattering matrix \( S_{LM}(d, \omega) \), a numerical method is employed to model \( S_{LM} \) as a function of frequency for every \( d \). Ansys HFSS is used to simulate \( S_{LM}(d, \omega) \) in the frequency range from 1 GHz to 5 GHz, for every \( d \in [0.03\lambda, 1.5\lambda] \) with steps of 0.01\( \lambda \). The Matrix-Fitting Toolbox of MATLAB is then used to find a rational function of frequency as suggested by the modeling recipe described in [12].

Figure 3 shows the plots of \( b_e \) and \( b_o \), in (17) and (18) as a function of distance \( d \). The dashed line represents the bound \( b_{iso} \), obtained when the dipoles are isolated. The bandwidth of the isolated antennas \( W_{iso} \) is obtained as in (21). The bounds (bandwidth) of the even and odd modes oscillate around the bound (bandwidth) value for isolated dipoles. For most \( d \), the red and blue curves are on opposite sides of the dashed horizontal line. Hence, the bandwidth of the overlapping region \( W_{ov} \) is almost always less than or equal, to \( W_{iso} \).

To compute the capacity (26), the noise spectral density is set to \( N_0 = -174 \text{ dBm/Hz} \). The threshold \( \tau \) is set to -50 dB such that the bandwidths obtained from Figure 3 are no
The capacity expression in (26) is then plotted for every \( d \) and shown in Figure 1. Figure 1 reveals the phenomenon that, even when \( W_{ov} \) is smaller than \( W_{iso} \), the capacity of the coupled system is higher than for isolated dipoles. The maximum capacity is achieved at \( 0.23\lambda \), which is 35% higher than isolated dipoles. For higher SNR, the gains can be more significant, with about 54% capacity increase for 20 dB at \( 0.23\lambda \). This boost in capacity for coupled dipoles is due to the increased matching bandwidth attained at \( d = 0.23\lambda \).

To examine the implications of operating the antennas over a wide band in a communication system with a receiver in the far-field, the dipoles are excited in even and odd mode at 1 and 5 GHz. The radiation patterns are then obtained using Ansys HFSS with spacing \( d = 0.23\lambda \). The three-dimensional radiation patterns for both modes are shown in Figure 4. Figures 4a and 4b show the even mode pattern transition from an omni-directional pattern at 1 GHz, to having a null towards the end-fire of the antenna array. This is in contrast with the odd mode in Figures 4c and 4d, which consistently exhibits a directional pattern with a null occurring towards the broad-side of the antenna configuration. Moreover, at 5 GHz, the beam of the even-mode is much narrower than that of the odd-mode.

The patterns demonstrate that the directivity of the coupled antennas is a strong function of frequency. Hence, even with small delay-spread we can expect that the channel between the transmitter and receiver to be frequency-selective. The implications of this for a communication system need further exploration.

V. CONCLUSION

The capacity of a simple coupled system of dipoles was analyzed, showing that coupling can lead to significant increase of capacity compared to an equivalent uncoupled system. It was shown that this increase was due to the high matching bandwidth that can be achieved for a coupled system.

REFERENCES