Can Operators Simply Share Millimeter Wave Spectrum Licenses?

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***Abstract*—Because of their often noise-limited behavior, millimeter wave systems may be able to share spectrum licenses without any coordination. We establish the theoretical feasibility of uncoordinated sharing by considering a downlink system containing multiple mmWave cellular providers. We compute the downlink rate distribution, and compare that against systems with exclusive licenses. We show that shared licenses can use a smaller bandwidth to achieve the same per-user median rate as providers with an exclusive spectrum license. We also show that both total interference and available bandwidth increase with the size of the spectrum sharing coalition, which implies that the optimal amount of spectrum sharing depends on the target rate.**

I. Introduction

Due to scarcity of spectrum at conventional cellular frequencies, the use of higher frequencies such as mmWave has been proposed for 5G cellular networks [1]–[3]. Communication at mmWave frequencies has non-trivial differences when compared to communication at conventional frequencies, *e.g.* use of highly directional antennas [4], and hence it causes less interference to neighboring BSs operating in the same frequency bands [1], [5]. This leads to the possibility of a new way of sharing spectrum licenses between independent cellular operators, possibly without any coordination.

Cellular networks are typically deployed by two or more independent cellular operators, distinguished by a closed access paradigm which allows only a given operator's customers to connect to its BSs under its exclusively licensed spectrum. This prevents the subscribers of other operators from interfering with their customers' transmissions. In millimeter wave systems, exclusive licenses may be wasteful in terms of spectrum usage. Due to the directionality of transmission and short propagation distances, at any given time and location, there is likely to be a very low level of interference. It is important to understand if and how spectrum licenses might be shared among different service providers to reduce the licensing costs and increase the utilization of spectrum.

Various cognitive license sharing schemes such as licensed shared access (LSA) and authorized shared access (ASA) were proposed [6], [7] to allow more than one entity to use the spectrum. As there are incumbent services, the above mentioned techniques [6], [7] would authorize a cellular system to transmit, only when the incumbent services are idle [8]. Implementation would require some kind of sensing or central coordination or use of a central database which keeps track of transmission of each licensee [9] to resolve the transmission conflicts, which may waste important resources in sensing/coordination/feedback. A mathematical model is required to derive insights to properly implement license sharing in mmWave systems. In recent work, stochastic geometry has emerged as an analytical tool to model and analyze a large variety of wireless communication systems. Most relevant here, the performance of a single operator mmWave system – accounting for effects such as blocking and antenna directionality – has been investigated in [5], [10], [11]. This prior work assumes the existence of a single operator, and analyzes the system over a single frequency band. To study the impact of having multiple operators sharing spectrum licenses amongst them, a more general framework that models these multiple operators is needed. A numerical framework was presented for a mmWave heterogeneous network with multiple tiers in [12]. The BSs, however, were assumed to have open access to all users, which is idealized as compared to the scenario envisioned in this paper, where each operator generally only allows its own subscribers to connect to its network.

In this paper, we establish the feasibility of uncoordinated sharing of spectrum licenses among cellular mmWave operators. We model a multi-operator mmWave system where every operator owns a spectrum license of fixed bandwidth with a provision to share the complete rights over its licensed spectrum with other operators. Next, we compute the performance of such system in terms of signal-to-interference-and-noise (SINR) and rate coverage probability using tools from stochastic geometry and show that spectrum license sharing achieves higher performance in terms of per user rate. We also consider the case where BSs of different networks are co-located to show that multiple networks can still share BS infrastructure while sharing the spectrum licenses. We investigate the effect of antenna beamwidth on the feasibility of license sharing and show that spectrum licenses sharing is more favorable as communication becomes more directional. Finally, we show that the optimal amount of spectrum sharing depends on the target rate for the system.

II. System Model

We consider a system consisting of M different cellular operators which coexist in a particular mmWave band. Each operator's network Φ_m consists of BSs whose locations are modeled using a Poisson Point Process (PPP) with intensity λ_m and users whose locations are distributed as independent PPP with intensity λ_m^{u}. The BSs of each operator can transmit with power P_m. We denote the total spectrum by B and suppose that the m^{th} operator owns a license for an orthogonal spectrum of B_m bandwidth which it can share with others.

We consider a typical user UE_0 at origin without loss of generality (thanks to Slivnyak's theorem [13]). Let us index the operator it belongs to by n. Now consider a link between this user and a BS of operator m located at distance x. This link can be LOS or NLOS link which we denote by the variable link type s, which can take values $s = \mathrm{L}$ (for LOS) or $s = \mathrm{N}$ (for NLOS). We assume that the probability of a link being LOS is dependent on x and independent of types of other links and is given by $p(x) = \exp(-\beta x)$ [5], [14]. The path loss from the BS to user is modelled as $\ell_s(x) = C_s \left(\|x\|\right)^{-\alpha_s}$ where α_s is the pathloss exponent and C_s is the gain for s type links. Let us denote the j^{th} BS of network m as B_{mj}. Hence the effective channel between BS B_{mj} and the user UE_0 is given as $h_{mj} P_m \ell_{s_{mj}}(x_{mj})$ where s_{mj} denotes the link type between the user and BS B_{mj} and h_{mj} is a exponential random variable denoting Rayleigh fading.

From the independent thinning theorem [13], the BS PPP of operator m can be divided into two independent (non-homogeneous) BS PPPs: a PPP containing all the BSs with LOS link to the user UE_0 , $\Phi_{m,\mathrm{L}}$ and a PPP containing all the BSs with NLOS link to the user UE_0 $\Phi_{m,\mathrm{N}}$. They have intensity $\lambda_{m,\mathrm{L}}(x) = \lambda_m p(x)$ and $\lambda_{m,\mathrm{N}}(x) = \lambda_m(1 - p(x))$, respectively. Note that this results in total $2M$ classes (known as tiers) of BSs where each tier is denoted by $\{m, s\}$. Here m and s represent the index of the operator and the link type, respectively.

We assume that BSs of every operator are equipped with a steerable antenna having radiation pattern given as [5]

$$G(\theta) = \begin{cases} G_1 & |\theta| < \theta_{\mathrm{b}} \\ G_2 & \text{otherwise} \end{cases}.$$

Here $G_1 \gg G_2$ and θ_{b} denotes half beamwidth. The angle between the BS B_{mj} antenna and direction pointing to the user UE_0 is denoted by θ_{mj}.

We assume that a user of operator n can be associated with any BS from a particular set of operators denoted by access set S_n. Two special cases of access are open and closed. In an open access system, a user can connect to any operator and therefore $S_n = \{1, 2, \cdots M\}$. In a closed access system, a user can connect only to the operator it belongs to, and therefore $S_n = \{n\}$.

We assume that license sharing is performed by forming mutually exclusive groups. All the operators in each group share the whole spectrum license such that each operator within a group has equal bandwidth available to it. The effective bandwidth available to each operator after sharing is denoted by W_m. The user UE_0 experiences interference from all operators operating in the spectrum of associated operator k. We denote this interfering set by Q_k which is the same as the sharing group containing k^{th} operator. Two extreme examples of license sharing are exclusive license and fully shared license. In the exclusive license scheme, each operator can use only its own license. Therefore the bandwidth available to each operator is $W_n = B_n$ and the interfering networks set is $Q_n = \{n\}$. In full sharing, all operators can use whole frequency band. Therefore the available bandwidth W_n to each operator is B and the interfering operator set is $Q_n = \{1, 2, \cdots M\}$.

Now, the effective received power from a BS B_{mj} at user UE_0 is given as

$$P_{mj} = P_j h_{mj} \ell_{s_{mj}}(x_{mj}) G(\theta_{mj}). \tag{1}$$

Hence, the average received power from B_{mj} at UE_0 without the antenna gain is given by

$$P_{mj}^{\mathrm{avg}} = P_m \ell_{s_{mj}}(x_{mj}). \tag{2}$$

We assume the maximum average received power based association in which any user associates with the BS providing highest P_{mj}^{avg} among all the operators it has access to (*i.e.* access set). Let us denote the operator the user UE_0 associates with by k and the index of the serving BS by i. Since the serving BS aligns its antenna with the user so that angle θ_{ki} between the serving BS antenna and user direction is 0^{o} and the effective received power of this BS is given as $P_{ki} = P_k h_{ki} \ell_{s_{ki}}(x_{ki}) G(0) = P_k h_{ki} \ell_{s_{ki}}(x_{ki}) G_1$. For each interfering BS B_{mj} for $m \in Q_k$, the angle θ_{mj} is assumed to be uniformly distributed between $-\pi$ and π. Now, the SINR at the typical user UE_0 of operator n at origin and associated with the i^{th} BS of operator k is given as

$$\mathrm{SINR}_{ki} = \frac{P_k h_{ki} \ell_{s_{ki}}(x_{ki}) G_1}{\sigma_k^2 + I} \tag{3}$$

where I is the interference from all BSs of operators in set Q_k and is given by

$$I = \sum_{m \in Q_k} \sum_{p \in \{\mathrm{L},\mathrm{N}\}} \sum_{j \in \Phi_{m,p}} P_m h_{mj} \ell_p(x_{mj}) G(\theta_{mj}). \tag{4}$$

The noise power for operator m is given by $\sigma_m^2 = N_0 W_m$ where N_0 is the noise power density. Since σ_m^2 is dependent on the allocated bandwidth, it varies accordingly with association.

III. SINR and Rate Coverage Probability

One metric that can be used to compare systems is the SINR coverage probability. It is defined as the probability that the SINR at the user from its associated BS is above a threshold T *i.e.* $\mathrm{P}^{\mathrm{c}}(T) = \mathbb{P}\left[\mathrm{SINR} > T\right]$, and is equivalently the CCDF (complementary cumulative distribution function) of the SINR. In this section, we will first investigate the association of a typical user of n^{th} operator to a BS and then compute the coverage probability for this user.

A. Association Criterion and Probability

Recall that the user UE_0 of the n^{th} operator can be associated with any operator from the set S_n. Let E_{ki} denote the event that the user is associated with the BS B_{ki} (*i.e.* the i^{th} BS of operator k). Let us denote the distance of this BS by $x = x_{ki}$ and type by $s = s_{ki}$ for compactness. The event E_{ki} is equivalent to the event that no other BS has higher P^{avg} at the user. This event can be further written as combination of following two events: (i) the event that no other BS of operator k has higher P^{avg} at the user, and (ii) that no BS of any other accessible operator m has higher P^{avg} at the user:

$$E_{ki} = \{P^{\mathrm{avg}}_{ki} > P^{\mathrm{avg}}_{kj}\ \forall i \neq j\} \cap \{P^{\mathrm{avg}}_{ki} > P^{\mathrm{avg}}_{mj}\ \forall m \in S_n \setminus \{k\}\}. \quad (5)$$

Using (2), it can be expressed as an equivalent condition over locations of all BSs (except the serving one) as follows:

$$E_{ki} = \left\{ x_{mj} > \left(\frac{P_m C_{s_{mj}}}{P_k C_s}\right)^{\frac{1}{\alpha_{s_{mj}}}} x^{\frac{\alpha_s}{\alpha_{s_{mj}}}}\ \forall\ m \in S_n \right\}.$$

where s_{mj} denotes the type of the link between UE_0 and BS B_{mj}. As seen from these condition, the average received power based association rule effectively creates exclusion regions around the user for BSs of each operator in S_n. Let us denote the exclusion radius of a tier $\{m, p\}$ by $D^{ks}_{mp}(x)$. For example, the exclusion region of all the LOS BSs of operator m when the user is associated with a NLOS BS of operator k is given by

$$D^{k\mathrm{N}}_{m\mathrm{L}}(x) = \left(\frac{P_m}{P_k}\frac{C_\mathrm{L}}{C_\mathrm{N}}\right)^{\frac{1}{\alpha_\mathrm{L}}} x^{\frac{\alpha_\mathrm{N}}{\alpha_\mathrm{L}}}. \quad (6)$$

This exclusion region denotes the region where interfering BSs cannot be located and hence, affects the sum interference. Note that for the BSs of the operators which are not in set S_n, there are no exclusion regions, *i.e.* $D^{ks}_{mp}(x) = 0\ \forall m \notin S_n$.

The probability that all BSs of tier $\{m, p\}$ are outside the exclusion radius d is given by the void probability of the PPP Φ_{mp} which is $\mu_{m,p}(d) = \exp(-\Lambda_{m,p}(\mathcal{B}(d)))$. Since the PPPs of the tiers are mutually independent, the probability density function of the distance x to this associated BS is given as

$$f_{k,s}(x) = 2\pi\lambda_{k,s} x\ \mu_{k,s}(x)\ \mu_{k,s'}\big(D^{ks}_{ks'}(x)\big) \prod_{m \in S_n \setminus \{k\}} \mu_{m,s}\big(D^{ks}_{ms}(x)\big)\mu_{m,s'}\big(D^{ks}_{ms'}(x)\big). \quad (7)$$

The probability that a user of network n is associated with a BS of operator k can be computed by summation over both LOS and NLOS tiers:

$$A^n_k = \int_0^\infty \left(f_{k,\mathrm{L}}(x) + f_{k,\mathrm{N}}(x)\right) dx. \quad (8)$$

Let $\mathrm{P}^\mathrm{c}_{k\mathrm{L}}$ and $\mathrm{P}^\mathrm{c}_{k\mathrm{N}}$ denote the probabilities of coverage for the typical user which is associated with a LOS and NLOS BS of operator k, respectively. They can be computed by integrating the CCDF of SINR from serving BS over pdf of distance x from serving BS as follows:

$$\mathrm{P}^\mathrm{c}_{ks} = \int_0^\infty \mathbb{P}\left[\mathrm{SINR}_{ks}(x) > T\right] f_{k,s}(x) dx = \int_0^\infty \mathbb{P}\left[P_k h_{ks} C_s G(0) > T(I + \sigma_k^2) x^{\alpha_s}\right] f_{k,s}(x) dx. \quad (9)$$

Since $h_{ks} \sim \exp(1)$, the probability in (9) can be replaced as

$$\mathrm{P}^\mathrm{c}_{ks} = \int_0^\infty \mathbb{E}\left[\exp\left(-\frac{T\sigma_k^2 x^{\alpha_s}}{C_s G_1 P_k} - \frac{T I x^{\alpha_s}}{C_s G_1 P_k}\right)\right] f_{k,s}(x) \mathrm{d}x = \int_0^\infty \exp\left(-\frac{T\sigma_k^2 x^{\alpha_s}}{C_s G_1 P_k}\right) \mathcal{L}_I\left(\frac{T x^{\alpha_s}}{C_s G_1 P_k}\right) f_{k,s}(x) \mathrm{d}x \quad (10)$$

where $\mathcal{L}_I(t)$ denotes the Laplace Transform of the interference I caused by BSs of all operators in set Q_k and is defined as $\mathcal{L}_I(t) = \mathbb{E}\left[e^{-tI}\right]$.

Since the association with different tiers are disjoint events, the SINR coverage probability of the typical user can be computed by summing these individual tier coverage probabilities over all accessible tiers:

$$\mathrm{P}^\mathrm{c}(T) = \sum_{k \in S_n} \mathrm{P}^\mathrm{c}_k(T), \quad (11)$$

where $\mathrm{P}^\mathrm{c}_k(T)$ is the sum probability of coverage over both tiers of operator k and is defined as

$$\mathrm{P}^\mathrm{c}_k(T) = \mathrm{P}^\mathrm{c}_{k\mathrm{L}}(T) + \mathrm{P}^\mathrm{c}_{k\mathrm{N}}(T). \quad (12)$$

To proceed further, we need to first characterize the interference I for which we will compute its Laplace Transform.

B. Interference Characterization

Due to mutual independence of the tiers, the Laplace transform of the interference (given by (4)) can written as product of the following terms:

$$\mathcal{L}_I(t) = \prod_{m \in Q_k} \mathcal{L}_{I_m}(t) = \prod_{m \in Q_k} \left(\mathcal{L}_{I_{m\mathrm{L}}}(t)\mathcal{L}_{I_{m\mathrm{N}}}(t)\right) \quad (13)$$

where $\mathcal{L}_{I_m}(t)$ refers to the interference caused by operator m and $\mathcal{L}_{I_{m\mathrm{L}}}(t)$ and $\mathcal{L}_{I_{m\mathrm{N}}}(t)$ denote the Laplace transforms of LOS and NLOS interference from operator m which are given in the following Lemma.

Lemma 1. *The Laplace transforms of the interference from LOS and NLOS BSs of operator m to a user of operator n which is associated with s type BS of operator k in a multi-operator system are given as*

$$\mathcal{L}_{I_{mp}}(t) = \exp\left(-2\lambda_m[\theta_\mathrm{b} F_p(\beta, \alpha_\mathrm{L}, tG_1 P_m C_p, D^{ks}_{mp}(x)) + (\pi - \theta_\mathrm{b}) F_p(\beta, \alpha_p, tG_2 P_m C_p, D^{ks}_{mp}(x))]\right)$$

where $F_\mathrm{L}(b, a, A, x) = \int_x^\infty \frac{e^{-by} A y^{-a}}{1 + A y^{-a}} y \mathrm{d}y,$

and $F_\mathrm{N}(b, a, A, x) = \int_x^\infty (1 - e^{-by}) \frac{A y^{-a}}{1 + A y^{-a}} y \mathrm{d}y.$

Proof: See Appendix A ■

Note that the term containing G_1 and θ_b denotes the interference from aligned BSs whose antennas are directed towards the considered user while the term containing G_2 and $(\pi - \theta_\mathrm{b})$ represents the interference from the unaligned BSs.

Now we provide the final expression for SINR coverage probability.

Theorem 1. *The SINR coverage probability of a typical user of operator n in a multi-operator system is given as*

$$\mathrm{P}^\mathrm{c} = \sum_{k\in S_n} \sum_{s\in\{\mathrm{L},\mathrm{N}\}} \int_0^\infty \prod_{m\in Q_k} \mathcal{L}_{I_{m\mathrm{L}}}\left(\frac{Tx^{\alpha_s}}{C_s G_1 P_k}\right) \mathcal{L}_{I_{m\mathrm{N}}}\left(\frac{Tx^{\alpha_s}}{C_s G_1 P_k}\right) \exp\left(-\frac{\sigma_k^2 T x^{\alpha_s}}{C_s G_1 P_k}\right) f_{k,s}(x)\mathrm{d}x \quad (14)$$

where $\mathcal{L}_{I_{mp}}(t)$ is computed in Lemma 1 and $f_{k,s}(x)$ is given as (7).

Proof: Substituting the value of $\mathcal{L}_I(t)$ from Lemma 1 in (11), we get the result. ■

In (14), the first summation is over all operators which UE_0 can connect to, weighted by the association probability. This weighing is included inside the term $f_{k,s}(x)$.

C. Rate Coverage

While the SINR shows the serving link quality, the rate represents the data bits received per second per user and hence is more realistic indicator of the system performance. In this section, we derive the downlink rate coverage which is defined as the probability of the rate of a typical user being greater than the threshold ρ, *i.e.* $\mathrm{R}(\rho) = \mathbb{P}[\mathrm{Rate} > \rho]$.

Let us assume that O_k denote the time-frequency resources allocated to each user associated with the 'tagged' BS of operator k. Therefore the instantaneous rate of UE_0 is given as $R_{ki} = O_k \log(1 + \mathrm{SINR}_{ki})$. The value of O_k depends upon the number of users (N_k^u), equivalently the load, served by the tagged BS. Similar to [11], [15], we take the mean approximation of the load which is modeled as follows:

$$N_k^\mathrm{u} = 1 + 1.28 \frac{1}{\lambda_k} \sum_{m:k\in S_m} \lambda_m^\mathrm{u} A_k^m. \quad (15)$$

Note that the summation is over all the operators whose users can connect to the operator k and the sum denotes the combined density of associated users from each operator.

Now we assume that the scheduler at the tagged BS gives $1/N_k^\mathrm{u}$ fraction of resources to each of the N_k^u users. Using the mean load approximation, the instantaneous rate of a UE_0 is given as

$$R_{ki} = \frac{W_k}{N_k^\mathrm{u}} \log(1 + \mathrm{SINR}_{ki}). \quad (16)$$

Let $\mathrm{R}_k^\mathrm{c}(\rho)$ denote the rate coverage probability when user is associated with operator k. Then the total rate coverage will be equal to sum of $\mathrm{R}_k^\mathrm{c}(\rho)$'s over all accessible operators:

$$\mathrm{R}^\mathrm{c}(\rho) = \sum_{k\in S_n} \mathrm{R}_k^\mathrm{c}(\rho). \quad (17)$$

Now $\mathrm{R}_k^\mathrm{c}(\rho)$ can be derived in terms of SINR coverage probability as follows:

$$\mathrm{R}_k^\mathrm{c}(\rho) = \mathbb{P}[R_{ki} > \rho] = \mathbb{P}[W_k/N_k^\mathrm{u} \log(1 + \mathrm{SINR}_{ki}) > \rho] = \mathrm{P}_k^\mathrm{c}\left(2^{\rho N_k^\mathrm{u}/W_k} - 1\right).$$

Therefore the rate coverage is given as

$$\mathrm{R}^\mathrm{c}(\rho) = \sum_{k\in S_n} \mathrm{P}_k^\mathrm{c}\left(2^{\rho N_k^\mathrm{u}/W_k} - 1\right). \quad (18)$$

IV. Performance Comparison

We use our mathematical framework to compare the benefits of spectrum licensing. We enumerate four specific cases (or systems) considering different combinations of accesses and license sharing schemes.

System 1: Status Quo: System 1 has closed access and exclusive licenses for each operator. This case is equivalent to a set of M single-operator systems which has been studied in prior work [5]. This system serves as a baseline case to evaluate benefits of sharing.

System 2: Sharing Utopia: System 2 has open access and fully shared licenses for each operator. The spectrum accessible to each operator W_k is the complete band B. Since users can connect to any operator, it requires full coordination among the operators including sharing of control channel and other resources. Therefore such system serves as an upper bound to the other two more practical systems. Note that if all M operators are identical with respect to every parameter, then this system is equivalent to a single-operator system with the aggregate BS and UE density.

System 3: Spectrum Sharing: System 3 has closed access and fully shared licenses for each operator. This case does not require any transmission coordination among networks or common control channel, nor does it require sharing of infrastructure or back-haul resources. This system is close to the practical implementation where subscribers must connect to their respective service providers only.

System 4: Co-located Sharing: System 4 has closed access and fully shared licenses for each operator where the respective BSs of all the operators are all co-located. This system will help us understand if independent operators can still share BS infrastructure while sharing the spectrum licenses. The system model for this case remains the same as the previous three systems except for the following two differences: 1) The BS locations are modeled by a single PPP $\Phi = \{x_j\}$ with intensity λ and 2) for a typical user, the BSs of all the operators located at the same location are either all LOS or all NLOS. We first briefly show the computation the probability of SINR coverage of this system. The BS PPP Φ can be divided in to two independent PPP, Φ_L and Φ_N with intensity $\lambda_\mathrm{L}(x) = \lambda p(x)$ and $\lambda_\mathrm{N}(x) = \lambda(1-p(x))$. The probability density function of the distance x of the associated BS of operator n is given as

$$f_s(x) = 2\lambda_s \pi x \exp\left(-\Lambda_s(\mathcal{B}(x))\right) \exp\left(-\Lambda_{s'}\left(\mathcal{B}\left(D_{s'}^s(x)\right)\right)\right) \quad (19)$$

where the exclusion radius $D_p^s(x)$ is the same for BSs of all the operators and given as

$$D_p^s(x) = \left(\frac{C_p}{C_s}\right)^{\frac{1}{\alpha_p}} x^{\frac{\alpha_s}{\alpha_p}}. \qquad (20)$$

The interference I at UE_0 from BSs of all the operators is given as

$$I = x^{-\alpha_s} C_s \sum_{m \in Q_n \setminus \{n\}} P_m h_{mi} G(\theta_{mi}) + \sum_{p=\mathrm{L,N}} \sum_{j \in \Phi_p \setminus \{i\}} x_j^{-\alpha_p} C_p \sum_{m \in Q_n} P_m h_{mj} G(\theta_{mj}). \qquad (21)$$

The following Lemma characterizes the Laplace Transform of the interference in (21) in the co-located BSs case.

Lemma 2. *The Laplace Transform of interference to a typical user of operator n with closed access which is associated to a BS of type s in a multi-operator system with co-located BSs is given as*

$$\mathcal{L}_I(t) = \prod_{m \in Q_n \setminus \{k\}} \left(\frac{\theta_\mathrm{b}/\pi}{1 + t x^{-\alpha_s} C_s P_m G_1} + \frac{(\pi - \theta_\mathrm{b})/\pi}{1 + t x^{-\alpha_s} C_s P_m G_2} \right) \times \prod_{p=\mathrm{L,N}} \exp\left(-2\pi\lambda \int_{D_p^s(x)}^{\infty} p(y) \left(1 - \prod_{m \in Q_n} \left(\frac{\theta_\mathrm{b}/\pi}{1 + t y^{-\alpha_p} C_p P_m G_1} + \frac{(\pi - \theta_\mathrm{b})/\pi}{1 + t y^{-\alpha_p} C_p P_m G_2} \right) \right) y \mathrm{d}y \right)$$

Proof: See Appendix B. ■

Similar to previous subsections, the coverage probability of the typical user is given as

$$\mathrm{P^c}(T) = \sum_{s=\mathrm{L,N}} \int_0^\infty \exp\left(-\frac{T N_0 W_n x^{\alpha_s}}{C_s G(0) P_n} \right) \mathcal{L}_I \left(\frac{T x^{\alpha_s}}{C_s G(0) P_n} \right) f_s(x) \mathrm{d}x \qquad (22)$$

where $\mathcal{L}_I(t)$ is given in Lemma 2 and $f_s(x)$ is given in (19). Since full sharing of license is assumed for this system, the spectrum accessible to each operator is B.

V. Numerical Results

In this section, we provide numerical results and compare the four aforementioned systems to provide insights for license sharing. For these numerical results, we consider a system consisting of two cellular operators with identical parameters, each operating in mmWave frequency (28 GHz bands) with BS intensity $30/\mathrm{km}^2$ which is equivalent to average cell radius of 103 m. Each operator has user density of $200/\mathrm{km}^2$. We have assumed $\beta = 0.007$ for blockage which has an average LOS region of 144 m. The transmit power is assumed to be 26dBm. The pathloss exponents for LOS and NLOS are $\alpha_\mathrm{L} = 2, \alpha_\mathrm{N} = 4$ and the corresponding gains are $C_\mathrm{L} = -60\mathrm{dB}, C_\mathrm{N} = -70\mathrm{dB}$. The total system bandwidth is 200 MHz with each operator having a license for 100 MHz.

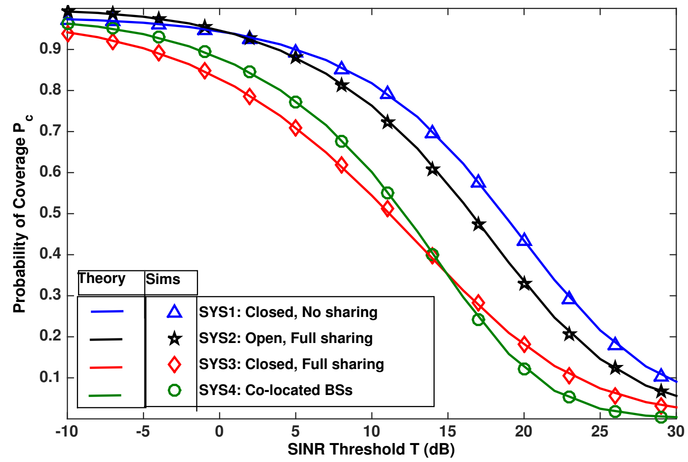

Fig. 1. Probability of SINR coverage in a two-operator mmWave system with BS antenna half beamwidth $\theta_\mathrm{b} = 10^\mathrm{o}$ for different cases. Line-curves denote values from the analysis and markers denote respective values from simulation.

SINR coverage trends: Fig. 1 compares the probability of SINR coverage for these systems. We can observe that the typical user in System 2 has high SINR coverage due to its open access. The closed access in System 3 allows BSs of another networks to be located closer than the serving BS and may lead to large interference. Therefore the typical user in System 3 has low SINR coverage. In the baseline system, System 1, the user faces no interference from other networks and hence the SINR coverage is greater than System 3. System 4 has similar values and trends as System 3 when compared to other systems. Due to co-location of BSs, System 4 always guarantees that no other operator's BS can provide higher received power than the serving BS for any user which is not true for System 3. But due to the same reason, there are always $K - 1$ interfering BSs of the other operators at the same location in System 4, while in System 3, that is not the case. Therefore we see a trade-off between System 3 and System 4 where for high values of SINR thresholds, SINR of System 3 is better, while System 4 performs better for low SINR thresholds.

Sharing licenses achieves higher rate coverage: Fig. 2 compares the probability of rate coverage for four systems which incorporates the effect of load and bandwidth. Since each operator has a large bandwidth and large SINR coverage in System 2, its rate coverage is the highest among all systems. Here we can see that even though System 1 has higher SINR coverage than System 3 (and 4), the latter achieves higher median rate, due to the extra bandwidth gained from spectrum license sharing. In particular, System 3 and 4 have respectively 25% and 32% higher median rates than System 1.

Impact of beamwidth on median rate: Fig. 3 compares the median rate of the four systems for various values of half-beamwidth. It can be seen that below a certain threshold for the half-beamwidth, sharing is optimal. For the given parameters, the threshold is at about 25^o. Since mmWave has typical half-beamwidth less than 15^o, sharing should increase the

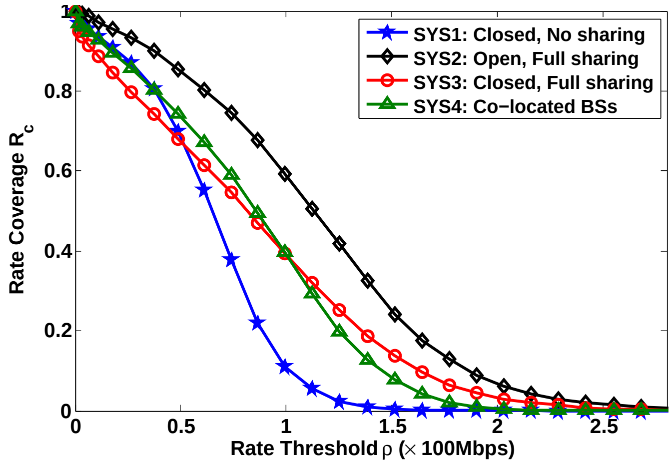

Fig. 2. Rate coverage in a two-operator mmWave system with BS antenna half beamwidth $\theta_b = 10^o$ for different cases. Systems 3 and 4 with shared license perform better than System 1 with exclusive licenses.

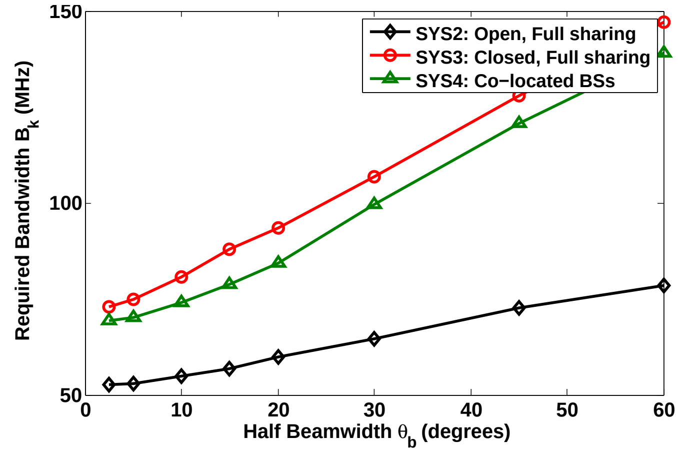

Fig. 4. Required bandwidth for each operator (with sharing of licenses) to achieve the same median rate achieved by the network with exclusive of spectrum with each network having 100MHz spectrum license. Sharing can reduce the license cost by more than 25%

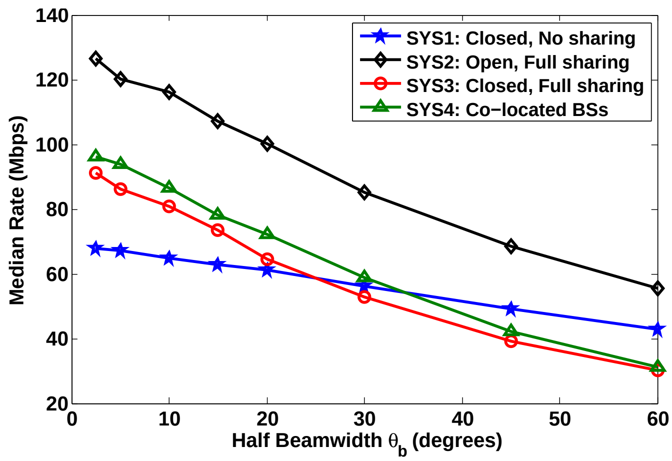

Fig. 3. Median rate versus BS antenna beamwidth in two-operator mmWave system under different cases. Systems with sharing of license outperforms System 1 with exclusive license for moderate and low values of antenna beamwidth.

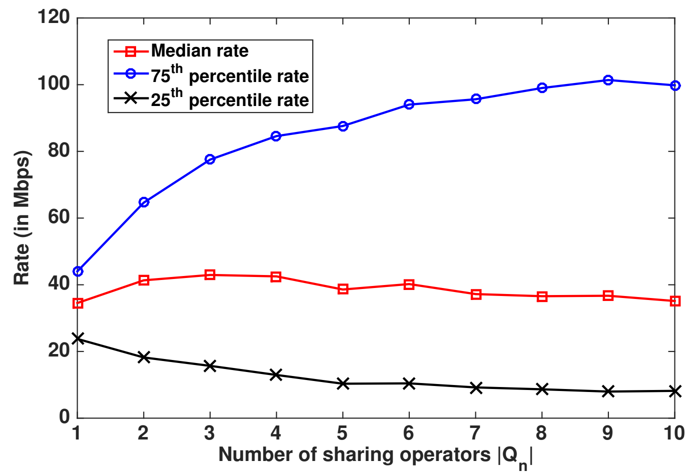

Fig. 5. Rate versus number of sharing operators in a mmWave cellular system with 10 operators. A trade-off between increasing the available bandwidth and increasing interference is observed.

achievable rate.

Sharing reduces spectrum cost significantly: Now we compare the following two cases. In the first case, each network owns a 100 MHz bandwidth exclusive license. This case is the same as System 1. In the second case, the networks share licenses completely and choose to buy just enough spectrum to achieve the same median rate as in the first case. Fig. 4 shows this required spectral bandwidth for each network. With a 10^o beamwidth antenna, each network only needs to buy 75 MHz of bandwidth which would save 25% of the license cost assuming linear pricing of the spectrum.

Optimal cardinality of sharing groups depends on the target rate: Now we consider a system with 10 operators with 50MHz bandwidth each and closed access. Fig. 5 shows variation of the per-user rate for different percentiles with respect to cardinality of sharing group which is equal to the number of operators sharing licenses with network n. We can see that the 75^{th} percentile rate increases with $|Q_n|$ while the 25^{th} percentile rate decreases. For the median rate, we see an increase up to $|Q_n|$=3 and then the median rate deceases. This trade-off is due to the fact that as more operators share their licenses, the total available bandwidth and the sum interference both increase. It can be observed that depending on the target performance, the optimal number of networks that should share their licenses varies.

VI. Conclusions

We have modeled a mmWave multi-operator system and derived the SINR and per-user rate distribution. We show that license sharing among operators improves system performance by increasing per-user rate and hence it is economical for operators to share their spectrum. Since an increasing number of networks increases both the sum interference and bandwidth, the optimal cardinality of the sharing group will depend on the target rate. Future work could include investigating the effect of low user density causing partial loading of BSs. Another useful direction is to investigate how multi-antenna techniques such as multiplexing including hybrid beamforming affect the insights about license sharing.

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APPENDIX A
PROOF OF LEMMA 1

The sum interference from all LOS BSs of network m at UE_0 is given as

$$I_{m\mathrm{L}} = \sum_{j \in \Phi_{m,\mathrm{L}} \cap \bar{\mathcal{B}}\left(0, D^{ks}_{m\mathrm{L}}(x)\right)} h_{mj} \|x_{mj}\|^{-\alpha_{\mathrm{L}}} P_m C_{\mathrm{L}} G(\theta_{mj})$$

where $\bar{\mathcal{B}}(0,r)$ denotes the compliment of a ball of radius r located at origin. This is due to the fact that all BSs are located outside the radius $D^{ks}_{m\mathrm{L}}(x)$. Its Laplace Transform is given as $\mathcal{L}_{I_{m\mathrm{L}}}(t) = \mathbb{E}\left[\exp(-tI_{m\mathrm{L}})\right]$. Now using the PGFL of PPP [13] and the moment generating function (MGF) of exponentially distributed h, the Laplace Transform can be written as

$$\mathcal{L}_{I_{m\mathrm{L}}}(t) = \exp\left(-\lambda_m \int_{D^{ks}_{m\mathrm{L}}}^{\infty} p(y) \left(2\pi - \int_0^{2\pi} \frac{d\theta}{1 + tG(\theta)P_m C_{\mathrm{L}} y^{-\alpha_{\mathrm{L}}}}\right) y\mathrm{d}y\right).$$

Now integrating with θ and then using the definition of function $F_{\mathrm{L}}(\cdot)$, we get the value of $\mathcal{L}_{I_{m\mathrm{L}}}(t)$. The Laplace Transform of the interference from the NLOS BSs can be computed similarly.

APPENDIX B
PROOF OF LEMMA 2

The Laplace Transform of interference given by (21), can be computed as

$$\mathcal{L}_I(t) = \mathbb{E}_{h,\theta}\left[e^{-tx^{-\alpha_s} C_s \sum_{m \in Q_k \setminus \{k\}} P_m h_{mi} G(\theta_{mi})}\right] \times \prod_{p=\mathrm{L,N}} \mathbb{E}_{\Phi_p,h,\theta}\left[e^{-t\sum_{j \in \Phi_p \setminus \{i\}} x_j^{-\alpha_p} C_p \sum_{m \in Q_k} P_m h_{mj} G(\theta_{mj})}\right].$$

Now using the PGFL of PPP [13] and independence of h_{mi}'s and θ_{mi}'s, the Laplace Transform can be written as

$$\mathcal{L}_I(t) = \prod_{m \in Q_k \setminus \{k\}} \mathbb{E}_{h,\theta}\left[e^{-tx^{-\alpha_s} C_s P_m h_{mi} G(\theta_{mi})}\right] \times \prod_{p=\mathrm{L,N}} \exp\left(-2\pi\lambda \int_{D_p^s(x)}^{\infty} p(y) \, \mathbb{E}_{h,\theta}\left[1 - e^{-ty^{-\alpha_p} C_p \sum_{m \in Q_k} P_m h_m G(\theta_m)}\right] y\mathrm{d}y\right).$$

Now using the MGF of exponentially distributed h_{mi}'s in first product term and the independence of h_m's and their MGF in the second product term, we get

$$\mathcal{L}_I(t) = \prod_{m \in Q_k \setminus \{k\}} \mathbb{E}_\theta\left[\frac{1}{1 + tx^{-\alpha_s} C_s P_m G(\theta_{mi})}\right] \times \prod_{p=\mathrm{L,N}} \exp\left(-2\pi\lambda \int_{D_p^s(x)}^{\infty} p(y) \, \mathbb{E}_\theta\left[1 - \prod_{m \in Q_k} \frac{1}{1 + ty^{-\alpha_p} C_p P_m G(\theta_m)}\right] y\mathrm{d}y\right).$$

Using the fact that $G(\theta_m)$'s are discreet random variables with $\mathbb{P}\left[G(\theta_m) = G_1\right] = \theta_{\mathrm{b}}/\pi$ and $\mathbb{P}\left[G(\theta_m) = G_2\right] = 1 - \theta_{\mathrm{b}}/\pi$, the Laplace Transform can be further written to get the Lemma.

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