Abstract—This paper addresses the optimal frequency reuse for the transmission of multimedia layered source which needs unequal target error rates or spectral efficiencies in its bitstream. We first analyze the crossover point of the outage probability curves for the full and partial frequency reuses, and show that that the crossover point in signal-to-noise ratio is a strictly increasing function in spectral efficiency. Based on the analysis, we propose the employment of the unequal frequency reuses for the transmission of a sequence of multimedia scalable packets.

I. INTRODUCTION

The increasing demand for multimedia services has motivated intensive research on the cross-layer optimization [1]. This paper studies the optimal frequency reuse for the transmission of multimedia layered (or progressive) source which needs unequal target error rates or spectral efficiencies in its bitstream. First, we analyze the crossover point of the outage probability curves of the narrowband system with partial frequency reuse, and the wideband system with full frequency reuse. We show that, as the spectral efficiency of the system increases, the crossover point in signal-to-noise ratio (SNR) monotonically increases. This holds for the single-input single-output (SISO) system as well for the multiple-input multiple-output (MIMO) schemes such as orthogonal space-time block codes (OSTBC), and vertical Bell Labs space-time architecture (V-BLAST) with a zero-forcing receiver. Next, we consider an orthogonal frequency-division multiplexing (OFDM) system which is able to employ either full or partial frequency reuse for its blocks of subcarriers. Based on our analysis, we propose the adoption of the unequal frequency reuses for the transmission of a sequence of multimedia scalable packets in an OFDM system.

II. PRELIMINARIES

Let \( N_{\text{rf}} (\geq 1) \) denote the frequency reuse factor of the cellular system. The \( N_{\text{rf}} = 1 \) yields the full frequency reuse of the wideband OFDM system, and \( N_{\text{rf}} > 1 \) yields the partial frequency reuse of the narrowband system. Let \( W \) denote the bandwidth per user within a cell. Then, each user is able to use a bandwidth of \( W/N_{\text{rf}} \). The downlink transmissions in neighboring cells that use the same frequency band (i.e., the same subcarriers in OFDM systems) as the target cell are averaged at a user, and this yields the interference to a user. A neighboring cell that uses the same frequency band is referred to as an interfering cell. In this paper, we consider the hexagonal planar cellular system where a base station is assumed to be located in the center of each cell. For such a system, which is depicted in Fig. 1, the distance between the centers of the nearest cells (i.e., the nearest base stations) that use the same frequency band is given by \( 2d_{\text{cell}} \sqrt{N_{\text{rf}}} \) [2], where \( d_{\text{cell}} \) is the distance between the center and the edge of the cell as is shown in Fig. 1. From this, the distance between the user in a target cell and the base station of the \( i \) th nearest interfering cell can be expressed as

\[
D_{\text{user},i} = 2d_{\text{cell}} \sqrt{N_{\text{rf}}} + d_{\text{user},i}
\]

where \( d_{\text{user},i} \) is the distance parameter which specifies the location of the user in a target cell, from the viewpoint of the base station in the \( i \) th nearest interfering cell. The range of \( d_{\text{user},i} \) is given by \(-2d_{\text{cell}}/\sqrt{3} \leq d_{\text{user},i} \leq 2d_{\text{cell}}/\sqrt{3} \), which is determined by the size of the cell (refer to Fig. 1).

Suppose that a user is at a distance \( x \) from the base station. The signal transmitted from the base station is received at the user with an attenuation of a factor \( x^{-\alpha} \) in power. Also suppose that all the base stations in the cellular system transmit the signals with the same power \( P \). Then, from (1), the received power of the interference signal that comes from the \( i \)th nearest interfering cell can be expressed as

\[
I_i = \frac{P}{D_{\text{user},i}^2} = \frac{P}{d_{\text{cell}}^2} \cdot \frac{1}{(2\sqrt{N_{\text{rf}}} + r_{\text{user},i})^{\alpha}}
\]

where \( r_{\text{user},i} \) is the normalized distance parameter which is defined by \( r_{\text{user},i} = d_{\text{user},i}/d_{\text{cell}} \) with the range given by \(-2/\sqrt{3} \leq r_{\text{user},i} \leq 2/\sqrt{3} \). In a model where the interference signals come from only the nearest interfering cells, the power of total out-of-cell interference can be expressed as

\[
I = \sum_{i=1}^{N_{\text{cell}}} I_i = \frac{P}{d_{\text{cell}}^2} \sum_{i=1}^{N_{\text{cell}}} f_1(N_{\text{rf}}, r_{\text{user}}, \alpha)
\]

where \( N_{\text{cell}} \) denote the number of the nearest interfering cells and \( f_1(N_{\text{rf}}, r_{\text{user}}, \alpha) \) is defined by

\[
f_1(N_{\text{rf}}, r_{\text{user}}, \alpha) = \sum_{i=1}^{N_{\text{cell}}} \frac{1}{(2\sqrt{N_{\text{rf}}} + r_{\text{user},i})^{\alpha}} > 0
\]

where \( r_{\text{user}} = [r_{\text{user},1}, \ldots, r_{\text{user},N_{\text{cell}}}] \) is the normalized distance parameter vector, and the inequality follows from \( N_{\text{rf}} \geq 1 \) and \(-2/\sqrt{3} \leq r_{\text{user},i} \leq 2/\sqrt{3} \).

For the downlink of target cell, we consider a system with \( N_{\text{t}} (\geq 1) \) transmit and \( N_{\text{r}} (\geq 1) \) receive antennas communicating over a frequency flat fading channel. A space-time
codeword, $S = [s_1 \ldots s_T]$ of size $N_t \times T$ is transmitted over $T$ symbol durations through $N_t$ transmit antennas. The baseband equivalent model of the system, at the $k$th time symbol duration ($k = 1, \ldots, T$), is given by
\[ y_k = Hs_k + n_k \] (5)
where $s_k$ is the $N_t \times 1$ transmitted signal vector, $y_k$ is the $N_r \times 1$ received signal vector, and $n_k$ is an $N_r \times 1$ noise vector at the output of a matched filter. The variable $n_k$ is zero-mean complex additive white Gaussian noise (AWGN) with $\mathcal{C} \mathcal{N} \left( n_k, n^H_k \right) = \sigma_n^2 I_N, \delta(k-l)$, where $\cdot^H$ and $\delta(\cdot)$ denote Hermitian operation and Kronecker delta function, respectively.

The single-sided power spectral density of any element of $n_k$ is denoted by $N_0$. In (5), $H$ denotes the $N_r \times N_t$ channel matrix, whose entry $h_{ij}$ represents the complex channel gain between the $j$th transmit antenna and the $i$th receive antenna, and the entries are i.i.d complex Gaussian random variables with zero mean and unit variance. It is assumed that $H$ is random, but constant over $T$ symbol durations (quasi-static Rayleigh i.i.d. fading).

III. CROSSOVER ANALYSIS OF THE OUTAGE PROBABILITIES FOR THE FULL AND PARTIAL FREQUENCY REUSES

In this section, we analyze the behavior of the crossover point of the outage probability curves for the full and partial frequency reuses. We first consider the SISO and the single-input multiple-output (SIMO) systems (i.e., $N_r \geq N_t = 1$). For a SISO or a SIMO system, the received SINR (signal to interference and noise ratio) at the user, which is at a distance $x$ from the base station of a target cell, can be expressed as

\[
\text{SINR} = \frac{P_{\text{rn}} \|H\|_F^2}{N_0W N_t + \frac{P_{\text{d}}}{d_{\text{cell}}} f_1(N_t, r_{\text{user}}, \alpha)} \cdot \frac{1}{N_t} + \frac{\gamma \|H\|_F^2}{N_t} \cdot \frac{W}{N_t} + \gamma \beta \alpha \frac{f_1(N_t, r_{\text{user}}, \alpha)}{N_t} \] (6)

where $\| \cdot \|_F$ denotes the Frobenius norm, $W/N_t$ is the bandwidth assigned to each user, $\gamma = P/x^\alpha N_0 W (> 0)$ is the received SNR at the user, and $\beta = x/d_{\text{cell}}$ is the user’s normalized distance from the base station. From Fig. 1, it is seen that the range of $\beta$ is given by $0 < \beta \leq 2\sqrt{3}$. From (6), the outage probability is given by [3]

\[
P_{\text{out}}(\gamma) = \text{Pr} \left[ W_{\text{rf}} - \frac{\gamma \|H\|_F^2}{N_t} \frac{W}{N_t} + \gamma \beta \alpha f_1(N_t, r_{\text{user}}, \alpha) < R \right] \] (7)

where $R$ is the transmission data rate of the user (bits/s). Using the cumulative density function (CDF) of $\|H\|_F^2$, a chi-square random variable with $2N_t (\geq 2)$ degrees of freedom for a SISO or a SIMO system, it can be shown that $P_{\text{out}}(\gamma)$ is given by (8) as is shown at the top of the next page.

For a wideband system with full frequency reuse, we set $N_t = 1$ in (8). Then, the outage probability of the wideband system, denoted by $P_{\text{out},w}(\gamma)$, is given by (9) as shown at the top of the next page, where $r_{\text{w}} = [r_{\text{w},1}, \ldots, r_{\text{w},N_{\text{cell}}}]$ is the normalized distance parameter vector, which is defined below (4), for the wideband cellular system. For a narrowband system with partial frequency reuse, we set $N_t = n(> 1)$ in (8). Then, the outage probability of the narrowband system, denoted by $P_{\text{out},n}(\gamma)$, is given by (10) as shown at the top of the next page, where $r_{\text{n}} = [r_{\text{n},1}, \ldots, r_{\text{n},N_{\text{cell}}}]$ is the normalized distance parameter vector for the narrowband cellular system with the reuse factor of $n(> 1)$.

Using (9) and (10), we find the SNR, $\gamma^*$, for which $P_{\text{out},w}(\gamma)$ and $P_{\text{out},n}(\gamma)$ are identical:

\[
P_{\text{out},w}(\gamma^*) = P_{\text{out},n}(\gamma^*) \text{ for } \gamma^* > 0 \] (11)

Through some steps, it can be shown that the existence of the crossover point in SNR, $\gamma^*$, is summarized as follows:
1) $\gamma^*$ is given by

\[
\gamma^* = \frac{1}{n} \left( \sum_{k=1}^{n} \frac{\beta \alpha f_1(1, r_{\text{w},k}, \alpha)}{\frac{f_1(n, r_{\text{n},1}, \alpha)}{\sum_{k=1}^{n} \frac{2}{\beta \alpha f_1(n, r_{\text{n},k}, \alpha)}}} \right) - 1
\] (12)

and $\gamma^* > 0$ if and only if $\sum_{k=1}^{n} \frac{\beta \alpha f_1(1, r_{\text{w},k}, \alpha)}{\frac{f_1(n, r_{\text{n},1}, \alpha)}{\sum_{k=1}^{n} \frac{2}{\beta \alpha f_1(n, r_{\text{n},k}, \alpha)}}} < f_1(1, r_{\text{w},k}, \alpha)/f_1(n, r_{\text{n},1}, \alpha)$ or, equivalently, if and only...
\[ P_{\text{out}}(\gamma) = 1 - \exp \left( - \left( \frac{1}{\gamma N_{\text{rf}}} + \beta^\alpha f_1(N_{\text{rf}}, r_{\text{user}}, \alpha) \right) \left( 2^{\frac{N_{\text{rf}}}{\gamma N_{\text{rf}}} R - 1} - 1 \right) \right) \sum_{k=1}^{N_r} \frac{1}{(k-1)!} \left\{ \left( \frac{1}{\gamma N_{\text{rf}}} + \beta^\alpha f_1(N_{\text{rf}}, r_{\text{user}}, \alpha) \right) \left( 2^{\frac{N_{\text{rf}}}{\gamma N_{\text{rf}}} R - 1} \right) \right\}^{k-1} \]  

(8)

\[ P_{\text{out},w}(\gamma) = 1 - \exp \left( - \left( \frac{1}{\gamma} + \beta^\alpha f_1(1, r_{\text{user}}^w, \alpha) \right) \left( 2^{\frac{\gamma}{\gamma} R - 1} - 1 \right) \right) \sum_{k=1}^{N_r} \frac{1}{(k-1)!} \left\{ \left( \frac{1}{\gamma} + \beta^\alpha f_1(1, r_{\text{user}}^w, \alpha) \right) \left( 2^{\frac{\gamma}{\gamma} R - 1} \right) \right\}^{k-1} \]  

(9)

\[ P_{\text{out},n}(\gamma) = 1 - \exp \left( - \left( \frac{1}{\gamma n} + \beta^\alpha f_1(n, r_{\text{user}}^n, \alpha) \right) \left( 2^{\frac{\gamma n}{\gamma n} R - 1} - 1 \right) \right) \sum_{k=1}^{N_r} \frac{1}{(k-1)!} \left\{ \left( \frac{1}{\gamma n} + \beta^\alpha f_1(n, r_{\text{user}}^n, \alpha) \right) \left( 2^{\frac{\gamma n}{\gamma n} R - 1} \right) \right\}^{k-1} \]  

(10)

If \( R < R^* \), where \( R^* \) is the data rate which satisfies the following equality

\[ \sum_{k=1}^{n} 2^{\frac{(n-k)R^*}{W}} = f_1(1, r_{\text{user}}^w, \alpha) / f_1(n, r_{\text{user}}^n, \alpha) \]  

(13)

2) \( \gamma^* \) is given by (12), and \( \gamma^* < 0 \) if and only if \( \sum_{k=1}^{n} 2^{\frac{(n-k)R^*}{W}} > f_1(1, r_{\text{user}}^w, \alpha) / f_1(n, r_{\text{user}}^n, \alpha) \) (or, equivalently, \( R > R^* \)).

3) \( \gamma^* \) does not exist if and only if \( \sum_{k=1}^{n} 2^{\frac{(n-k)R^*}{W}} = f_1(1, r_{\text{user}}^w, \alpha) / f_1(n, r_{\text{user}}^n, \alpha) \) (or, equivalently, \( R = R^* \)).

Note that \( \gamma^* < 0 \) implies that the crossover point of \( P_{\text{out},w}(\gamma) \) and \( P_{\text{out},n}(\gamma) \) does not exist in the range of \( \gamma > 0 \).

Moreover, through some steps, it can be shown that the crossover point in SNR, \( \gamma^* \), is a strictly increasing function in \( R > 0 \) as long as \( \gamma^* > 0 \) holds (i.e., the crossover point exists in the range of \( \gamma > 0 \)). Note that this result is valid regardless of the location of the user in the target cell (i.e., \( r_{\text{user}}^w \) and \( r_{\text{user}}^n \)), path-loss exponent (\( \alpha > 0 \)), bandwidth (\( W > 0 \)), and the frequency reuse factor of the narrowband system (\( N_{\text{rf}} > 1 \)). Also note that the above result is effective for all SNR, since it is derived from the exact outage probabilities, \( P_{\text{out},w}(\gamma) \) and \( P_{\text{out},n}(\gamma) \), given by (9) and (10), respectively, without any high SNR approximate outage probabilities being used. Further, it can be proven that, for the V-BLAST with a zero-forcing receiver or OSTBC in a MIMO system, the crossover point in SNR exhibits the same behavior.

Next, we compare the outage probabilities, \( P_{\text{out},w}(\gamma) \) and \( P_{\text{out},n}(\gamma) \), for the full and partial frequency reuses, respectively. It can be proven that

- If \( R < R^* \), then \( P_{\text{out},n}(\gamma) < P_{\text{out},w}(\gamma) \) for \( \gamma > \gamma^* > 0 \)
- \( P_{\text{out},n}(\gamma) > P_{\text{out},w}(\gamma) \) for \( 0 < \gamma < \gamma^* \)  

(14)

Fig. 2. Outage probabilities of the full and partial frequency reuses for \( R_1 < R_2 \).

- If \( R \geq R^* \), then either \( \gamma^* < 0 \), or \( \gamma^* \) does not exist. For this case, we have

\[ P_{\text{out},n}(\gamma) > P_{\text{out},w}(\gamma) \]  

for \( \gamma > 0 \)  

(15)

Let \( \gamma^*_1 \) denote the crossover point when a transmission data rate \( R = R_1 \) is used, and let \( \gamma^*_2 \) denote the crossover point when a data rate \( R = R_2 \) is employed. Suppose that \( R_1 < R_2 \). Then, from the analysis above, we have \( \gamma^*_1 < \gamma^*_2 \). The outage probabilities are qualitatively depicted in Fig. 2 for the case of \( \gamma^* > 0 \). Suppose that SNR, \( \gamma \), is smaller than \( \gamma^*_2 \) but greater than \( \gamma^*_1 \). Then, from Fig. 2, it is seen that partial frequency reuse is preferable to the full frequency reuse for a data rate \( R_1 \), whereas the full frequency reuse is preferable for a data rate \( R_2 \) (\( R_1 < R_2 \)).

IV. Unequal Frequency Reuses for the Transmission of a Layered Bitstream

Based on the analysis in the previous section, we propose the adoption of the unequal frequency reuses for the transmission of the applications which need unequal target error
rates or spectral efficiencies in their bitstream. To begin, we briefly present the transmission of layered multimedia sources. Progressive encoders employ a mode of transmission so that encoded data have gradual differences of importance in their bitstreams [4]. Suppose that the system takes the bitstream from the progressive source encoder, and transforms it into a sequence of \( N_p \) packets [5]. We suppose that each of these \( N_p \) progressive packets can be encoded with different transmission data rates, as well as can be transmitted over the subcarriers with different frequency reuses, so as to yield the best end-to-end performance. The error probability of an earlier packet needs to be lower than or equal to that of a later packet, due to the gradually decreasing importance in the progressive bitstream. Thus, given the same transmission power, the earlier packet requires a transmission data rate which is lower than or equal to that of the later packet.

Let \( N_D \) denote the number of candidate data rates employed by a system. The number of possible assignments of \( N_D \) data rates to \( N_P \) packets would exponentially grow as \( N_P \) increases. Further, if each packet can be transmitted with full or partial frequency reuse, the selection of frequency reuses as well as data rates for \( N_P \) packets yields a more complicated optimization problem. Note that each source, such as an image, has its inherent rate-distortion characteristic, from which the performance of the expected distortion is computed. Hence, for example, when a series of images is transmitted, the above optimization should be addressed in an image-by-image manner (i.e., in a real time manner), considering which specific image (i.e., rate-distortion characteristic) is transmitted in the current time slot. To address this matter, there have been many studies about the optimal assignment of data rates for a sequence of progressive packets [6].

Suppose that the \( k \)th packet in a sequence of \( N_P \) packets is transmitted with full frequency reuse. Then, our analysis tells us that the \( k+1 \)st, \( k+2 \)nd, \ldots, \( N_P \)-th packets also should adopt full frequency reuse rather than partial one. This is because in Section III we have shown that, when full frequency reuse is preferable for a packet with a data rate of \( R_1 \), a packet with \( R_2 (\geq R_1) \) also should employ full frequency reuse, as long as the transmission power of the latter is the same as or smaller than that of the former (see Fig. 1). That is, in a sequence of \( N_P \) progressive packets, the last \( i \) consecutive packets should adopt full frequency reuse, and the other \( N_P - i \) packets should employ partial frequency reuse \((0 \leq i \leq N_P)\). Note that the strategy is based on the properties of progressive sources that are involved with unequal data rates or transmission powers in the bitstream.

V. NUMERICAL EVALUATION

The outage probabilities for the full and partial frequency reuses are numerically evaluated for the OSTBC scheme in \( 2 \times 3 \) MIMO channels with various spectral efficiencies. The results are shown in Fig. 3. It is seen that as the spectral efficiency increases, the crossover point of the outage probabilities behave in a manner as predicted by the analysis in Section III (see Fig. 1).

VI. CONCLUSIONS

The behavior of the crossover point of the outage probability curves for full and partial frequency reuses was analyzed in terms of the spectral efficiency. Based on this, we proposed the application of the unequal frequency reuses to the transmission of a sequence of progressive packets.

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