

Bit Allocation for Increased Power Efficiency in 5G Receivers with Variable-Resolution ADCs

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Abstract—In future mmWave wireless system, fully digital receivers may have an excessive power consumption at the Analog to Digital Converters (ADC), even if lower resolution ADCs are employed. We propose to optimize the ADC resolution exploiting the sparse propagation in mmWave. We identify and assign more bits to antennas that capture stronger incoming signals, and allocate fewer bits to the antennas that see mostly noise. In order to facilitate a potential practical implementation, we constrain the allocation problem so the number of bits assigned to each antenna can take only one of two values, b_{low} or b_{high} . Compared to a reference fixed-resolution mmWave system with b_{ref} bits ($b_{low} \leq b_{ref} \leq b_{high}$), and depending on the margin between the two options given to the algorithm, (b_{low}, b_{high}) , our results show that 2-level receivers with a low margin (e.g., (4, 6)) can achieve moderate power saving (5-20%) consistent across any received unquantized SNR value, whereas 2-level receivers with a wide margin (e.g., (1, 8)) can achieve a large power saving (80%) only at high SNR, while consuming more power than the reference at low SNR. Combining bit allocation with antenna selection techniques, we create a 3-level system (e.g., 0, 4, 8) that can outperform the former scenario when the given resolution options are carefully chosen.

Index Terms—Millimeter Wave, Massive MIMO, Digital Beamforming, Energy Efficiency, Antenna Selection, Low Resolution ADCs, Variable Resolution ADCs

I. INTRODUCTION

Future wireless communications are expected to leverage large antenna arrays at the base station to achieve higher data rates, both in new millimeter wave (mmWave) bands and at standard frequencies with massive multiple input multiple output (MIMO) [1], [2]. Fully digital receiver architectures, where each antenna is connected to an independent Analog to Digital Converter (ADC), can provide maximum flexibility but could display too high component power consumptions due to the exponential increase of ADC power with the number of bits [3]. The concept of green communication and the deployment of ultra-dense small cells motivate the reduction of power consumption at the base station.

There are two strategies to mitigate the power consumption of receivers with many antennas:

- 1) Use Analog or Hybrid Combining (AC or HC) to perform all or a part of the MIMO operations in analog

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circuitry and sample only one or a few signals with ADCs [4], [5].

- 2) Use fully Digital Combining (DC) with reduced ADC resolution (for example, 1 or a few bits), which can offer even better power efficiency if the power of radio-frequency (RF) components is taken into account [6]–[8].

In this work, focusing on an uplink scenario, we propose a further improvement to the fully-digital low-resolution strategy by studying the possibility of enabling a variable number of bits in each ADC of the DC system. Compared to a conventional approach to low-resolution DC, where each RF chain has equal ADCs with the same fixed number of bits b_{ref} , we propose assigning to some ADCs a slightly higher number of bits $b_{high} > b_{ref}$, while the rest of the RF chains have an even lower number of bits ($b_{low} < b_{ref}$). Our results show that the same achievable rate of the fixed-bit system can be achieved using two variable-bit values with a power saving between 20 and 80%, depending on the difference between the pair of resolution options (b_{low}, b_{ref}) and the link pre-quantization Signal to Noise Ratio (SNR).

A. Related Work

Recent works such as [9]–[11] study the achievable rate and energy efficiency (EE) of large antenna array receiver designs depending on the ADC resolution. The effect of the number of ADC bits b and the sampling rate B on achievable rate and power consumption is analyzed in [12] for both AC and DC.

DC systems using low-resolution ADCs to reduce power consumption are further analyzed in [7], [13], showing that a few bits are enough to achieve almost the spectral efficiency (SE) of an unquantized system of the same characteristics.

Antenna selection using analog switches is proposed as an alternative to phase shifters for HC implementation in [14]. In addition, a combination of switching and variable resolution is presented in [15], where the best antennas are connected to high resolution ADCs and the rest are connected to 1-bit ADCs. In this work we study a more generalized 2-value and antenna selection problem where the margin between b_{low} and b_{high} can be arbitrary, and show that in some cases not so high margins such as $b_{low} = 4$ vs $b_{high} = 6$ work better.

It must be noted that constraining the allocation to one of two values is not imperative. In fact works such as [16], [17]

have considered more general problems where the number of bits for each antenna can be selected from the full range of natural numbers, or even solved the relaxed problem over the set of real numbers and then rounded the solution to an integer value. We justify a 2-level variable resolution abstraction due to the fact that similar (3-level) variable resolution ADC circuit components are readily available in the literature [18], and the 2-level constraint relieves some computational complexity so that it will be more reasonable to recalculate the bit allocation every time the fast fading changes (in the order of milliseconds).

B. Our Contribution

We propose two different scenarios depending on whether ADCs can only take two resolution values, or can also be completely shut off performing antenna selection. Thus, we solve a 2-level variable bit allocation problem and a 3-level problem where the first level is always zero. We focus on studying how the availability of variable low resolution ADCs can reduce mmWave receiver power consumption while maintaining the same achievable rate as some given reference fixed-resolution receiver scheme with b_{ref} bits.

- We show a receiver allocation algorithm with 2-level ADC resolutions (b_{low}, b_{high}) that can achieve similar rate to a fixed resolution scheme. However, we compare with a higher rate reference than the similar work in [7], [13]. Instead of 2 bits like in [13] our reference is the fixed resolution with the best trade-off between SE and EE under the same channel model, which is $b_{ref} = 5$ according to the results in [19], [20].
- The power saving is related to the margin between the two levels of resolution given to the algorithm, b_{low} and b_{high} . Robust performance at low SNR is obtained for not-too-low b_{low} and not-too-high b_{high} , whereas at high SNR more power is saved with more extreme differences between b_{low} and b_{high} .
- We consider a receiver allocation algorithm with combined 2-level bit allocation and antenna selection, which may also be considered as a 3-level system where the lower level is always zero ($0, b_{low}, b_{high}$). We show that at high SNR the system performance depends on the pair of values ($0, b_{high}$) regardless of b_{low} and is more power efficient than the first algorithm, thanks to the additional option of antenna selection, whereas it behaves similarly to the first algorithm with (b_{low}, b_{high}) at low SNR. Thus, selecting moderately high b_{low} for robustness the second algorithm can reap most benefits in all the unquantized SNR range.
- In addition, the power saving increases slightly with a larger number of antennas, and so enabling variable resolution is even more interesting in massive MIMO systems.

II. SYSTEM MODEL

A. mmWave Channel

We study mmWave point-to-point uplink MIMO links with an N_t antenna transmitter, an N_r antenna receiver and

bandwidth B . We assume that there is no inter-symbol interference, as in previous models such as [21]. The received signal in each symbol period $1/B$ is

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (1)$$

where \mathbf{x} represents the transmitted symbol vector, \mathbf{n} is the independent and identically distributed (i.i.d) circularly symmetric complex Gaussian noise vector, $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, N_o\mathbf{I})$, where N_o represents the noise power, and \mathbf{H} represents the $N_r \times N_t$ channel matrix. The mmWave channel matrix \mathbf{H} is randomly distributed following a random geometry with a small number of propagation paths (order of tens) grouped in very few clusters of similar paths [21], and is obtained as

$$\mathbf{H} = \sqrt{\frac{N_t N_r}{\rho N_c N_p}} \sum_{k=1}^{N_c} \sum_{\ell=1}^{N_p} g_{k,\ell} \mathbf{a}_r(\phi_k + \Delta\phi_{k,\ell}) \mathbf{a}_t^H(\theta_k + \Delta\theta_{k,\ell}) \quad (2)$$

where the terms in this expression are generated according to the mmWave channel model in [21]. Here ρ is the pathloss, $g_{k,\ell}$ is the small scale fading coefficient associated with the ℓ^{th} path of the k^{th} cluster, \mathbf{a}_t and \mathbf{a}_r are the spatial signatures of the transmit and receive arrays, and the θ 's and $\Delta\theta$'s and ϕ 's and $\Delta\phi$'s are the angles of departure and arrival for a small number of propagation paths N_p grouped in even fewer independent clusters N_c .

It must be noted that, due to this small number of paths, despite having large dimensions, the matrix \mathbf{H} has a low rank and an even lower number of dominant eigenvalues are responsible for 95% of the energy transfer in the channel. \mathbf{H} is generated in [21] using $N_c \sim \text{Poisson}(1.8)$ and $N_p = 20$. In this paper, we generate \mathbf{H} instead with $N_c = 2$, $N_p = 10$ for exact compatibility with fixed-resolution power consumption values obtained in [19], where N_c is selected as a constant and varied to study its effect. Moreover, it is noted in [21] that for the median channel a single spatial dimension captures approximately 50% of the channel energy and two degrees of freedom capture 80% of the channel energy. We also performed our own Monte-Carlo verification with 10^4 channel realizations, and found that the first eigenvalue is responsible for over 50% energy transfer with probability 0.95 and for over 75% of the energy transfer with probability 0.6.

We exploit this property of the mmWave channel distribution to design an efficient transmitter MIMO precoding that is also tractable for bit allocation. We assume that the transmitter has channel state information (CSIT) and implements a beamforming scheme that concentrates all the signal in the single strongest eigenvalue of the channel matrix. That is, if $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$ is the Singular Value Decomposition of the channel with values $\sigma_1 > \sigma_2 \geq \dots \sigma_{\min(N_t, N_r)}$, the transmitter sends a scalar symbol, x , projected over the row \mathbf{v}_1 of \mathbf{V}^H associated with the strongest singular value σ_1 . Thus, the received signal is

$$\mathbf{y} = \mathbf{u}_1 \sigma_1 x + \mathbf{n} \quad (3)$$

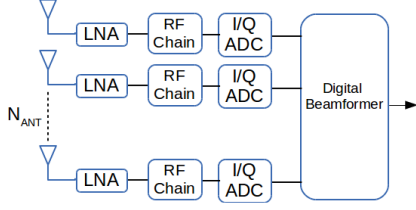


Figure 1. Digital Receiver with ADC with b_i bits on each antenna RF chain.

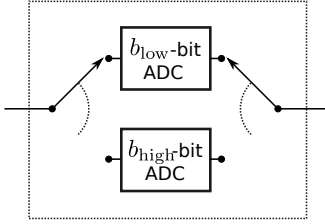


Figure 2. A simplified 2-level variable-resolution ADC.

where σ_1 is the maximum singular value, and \mathbf{u}_1 is the corresponding left singular vector.

B. Variable-bits ADC Receiver

The DC receiver is illustrated in Fig. 1. After the signal (1) is received, the signal at each antenna i is quantized by an ADC with b_i bits. Due to the fact that we are only concerned about power consumption in this paper, and ignore hardware complexity, we use the variable resolution ADC architecture abstraction in Fig. 2, consisting in a pair of fixed-resolution ADCs that can be alternatively switched in and out of the circuit. This architecture abstraction may be improved in practice as circuit designs such as [18] are introduced.

We represent the signal after quantization using the Additive Quantization Noise Model (AQNM) [12] approximation by adding an additive white noise \mathbf{n}_q that models the quantization distortion of each coefficient y_i of the signal (1), producing a quantized output in each ADC that satisfies

$$y_i^q = (1 - \eta_i)y_i + n_i^q \quad (4)$$

where η_i is the inverse of the signal-to-quantization noise ratio at antenna i , and is inversely proportional to the square of the resolution of the i -th ADC (i.e., $\eta_i \propto 2^{-2b_i}$). The quantization noise in each antenna n_i^q is AWGN distributed with variance $\eta_i(1 - \eta_i)\mathbb{E}[|y_i|^2]$. For a Gaussian input distribution, the values of η for $b \leq 5$ are listed in Table I, and for $b > 5$ can be approximated by $\eta = \frac{\pi\sqrt{3}}{2}2^{-2b}$ [13].

We can write the quantized signal as a vector by grouping all η_i 's in a diagonal matrix, producing

$$\mathbf{y}^q = \mathbf{D}(\mathbf{H}\mathbf{x} + \mathbf{n}) + \mathbf{n}^q, \quad \mathbf{D} = \begin{pmatrix} (1 - \eta_1) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & (1 - \eta_{N_r}) \end{pmatrix}, \quad (5)$$

where $\mathbf{D} = (1 - \eta_{\text{ref}})\mathbf{I}$ when the number of bits is the same in all ADCs. With appropriate modulation and coding

Table I
 η vs b FOLLOWING [13]

b	1	2	3	4	5
η	0.3634	0.1175	0.03454	0.009497	0.002499

schemes the achievable rate of the quantized MIMO link can approach

$$C_q = \mathbb{E}_{\mathbf{H}} \left[\max_{\mathbf{R}_{\mathbf{x}\mathbf{x}}} B \log_2 \left| \mathbf{I} + \frac{(1 - \eta)(\mathbf{H}\mathbf{R}_{\mathbf{x}\mathbf{x}}\mathbf{H}^H)}{N_o\mathbf{I} + \eta(\mathbf{H}\mathbf{R}_{\mathbf{x}\mathbf{x}}\mathbf{H}^H)} \right| \right], \quad (6)$$

for the general case of arbitrary \mathbf{x} . Since in this paper we exploit the single-dimensional nature of the mmWave channels matrix, we can replace $\mathbf{R}_{\mathbf{x}\mathbf{x}}$ with $|x|^2\mathbf{v}_m\mathbf{v}_m^H$, and the optimal receiver for this transmission is Maximum Ratio Combining so, if we denote by γ_q the SNR of \mathbf{y}_q , the achievable rate becomes

$$C_q = \mathbb{E}_{\mathbf{H}} [\log(1 + \gamma_q)] \quad (7)$$

where $\mathbb{E}[\cdot]$ represents the expectation and the SNR is computed as

$$\gamma_q = \sum_i \frac{\sigma_m^2 |u_m^i|^2 (1 - \eta_i)^2}{N_o(1 - \eta_i)^2 + (N_o + \sigma_m^2 |u_m^i|^2) \eta_i (1 - \eta_i)} \quad (8)$$

Note that under our chosen transmission scheme the maximization of rate can be achieved by simply maximizing the sum of independent per-antenna partial SNR contributions

$\gamma_q = \sum_i \gamma_q^i$, where $\gamma_q^i = \frac{\sigma_m^2 |u_m^i|^2 (1 - \eta_i)^2}{N_o(1 - \eta_i)^2 + (N_o + \sigma_m^2 |u_m^i|^2) \eta_i (1 - \eta_i)}$. Also, we can measure the unquantized SNR per antenna $\gamma_i = \frac{|u_m^i|^2 \sigma_m^2}{N_o}$ such that we may compute $\gamma_q^i = \frac{(1 - \eta_i)\gamma_i}{1 + \eta_i\gamma_i}$.

C. Initial SNR Measurement

We assume that channel state information of each receive antenna is perfectly known. In practice, an estimation of the SNR will be needed. The impact of imperfect initial SNR estimation is left as part of our future work. For example in [18], an ADC design is proposed where inputs with lower voltage use 6 bits and inputs with higher voltage use 4 bits. Such a design could be easily modified to operate in the opposite way, giving 6 bits to the signals with the higher voltage.

D. Digital Receiver Power Consumption

The devices required to implement the mmW receiver architecture are displayed in Fig. 1. All receiver schemes considered in this paper have the same RF components and we are only interested in the variation of ADC power consumption as a function of the number of bits.

The power consumption of the i -th ADC, denoted as $P_{ADC}^i = cB2^{b_i}$, increases exponentially with the number of bits b_i and linearly with the bandwidth B and with the ADC Walden's figure of merit c [22] (the energy consumption per conversion step per Hz). The aggregate power consumption across all the ADCs in the system is

$$P_{ADC}^{Tot} = \sum_i P_{ADC}^i = cB \left(\sum_i 2^{b_i} \right) \quad (9)$$

where it must be noted that we consider that all ADCs have the same Walden's figure of merit despite the variation of bits.

Algorithm 1 Greedy Bit Allocation

Reference effective SNR γ^{ref}
Measure unquantized SNR on each antenna γ^i
Order the RF chains as $\gamma^1 \geq \gamma^2 \geq \dots \gamma^{N_r}$
Start assuming $N_{\text{high}} = 0$, $b_i = b_{\text{low}} \forall i$
while $\sum_{i=1}^{N_r} \gamma_q^i < \gamma_q^{\text{ref}}$ **do**
 $N_{\text{high}} = N_{\text{high}} + 1$
 $b_i = b_{\text{high}}$
end while

III. ANALYSIS OF THE VARIABLE BIT ADC SYSTEM

Denoting the number of antennas with b_{high} bits by N_{high} , we write the normalized power consumption (ratio between the power consumption with variable and fixed resolution) as

$$\begin{aligned} \xi &= \frac{cB(N_{\text{high}}2^{b_{\text{high}}} + (N_r - N_{\text{high}})2^{b_{\text{low}}})}{cBN_r2^{b_{\text{ref}}}} \\ &= \frac{N_{\text{high}}}{N_r} 2^{b_{\text{high}} - b_{\text{ref}}} + \left(1 - \frac{N_{\text{high}}}{N_r}\right) 2^{b_{\text{low}} - b_{\text{ref}}} \end{aligned} \quad (10)$$

Regarding achievable rate, we have that η_i decreases exponentially to 0 as b_i grows, and each term γ_q^i in (8) increases monotonically as $\eta_i \rightarrow 0$. Thus, γ_q^i always increases with b_i and the increase in SNR obtained by increasing b_i is independent of the state of the other antennas. Moreover, since $\gamma_q = \sum_i \gamma_q^i$, the antennas with a higher coefficient γ_i produce the highest increase in received SNR when assigned more bits and if any pair of antennas i, j satisfies $\gamma_i > \gamma_j$ but $b_i < b_j$, the system can always get a higher rate if we swap b_i and b_j . The above ideas inspire Algorithm 1 (GBA).

The GBA algorithm starts with all ADCs in the low assignment, and swaps to a higher number of bits one antenna at a time until the system has the same effective SNR as the reference. If we wish for the variable-bit system to consume less, or equal power as the reference system, we must have

$$N_{\text{high}} \leq \frac{2^{b_{\text{ref}} - b_{\text{low}}} - 1}{2^{b_{\text{high}} - b_{\text{low}}} - 1} N_r \quad (11)$$

and thus two outcomes are possible from GBA: if at the end of the algorithm the number of high-resolution RF chains is $N_{\text{high}} < \frac{2^{b_{\text{ref}} - b_{\text{low}}} - 1}{2^{b_{\text{high}} - b_{\text{low}}} - 1} N_r$, power has been saved by GBA. Otherwise, GBA wastes more power than the reference scheme. We correct this shortcoming in Algorithm 2 (Greedy Antenna Selection and Bit Allocation, GASBA).

In the GASBA algorithm, three values are allowed per ADC, 0, b_{low} or b_{high} bits. The algorithm combines the mechanics of antenna selection and bit allocation, and always ensures that the variable resolution ADCs architecture does not exceed the power consumption of the reference fixed-bit ADCs by limiting the maximum number of b_{high} ADCs (according to Eq. (11)). However, depending on the operating SNR regimes, GASBA may result in a slight rate reduction.

IV. NUMERICAL SIMULATIONS

In this section we present the power savings achieved by GBA and GASBA algorithms, evaluated by Monte

Algorithm 2 Greedy Antenna Selection and Bit Allocation

Reference effective SNR γ^{ref}
Measure unquantized SNR on each antenna γ^i
Order the RF chains as $\gamma^1 \geq \gamma^2 \geq \dots \gamma^{N_r}$
Start assuming $N_{\text{on}} = 0$, $N_{\text{high}} = 0$, $b_i = 0 \forall i$
while $\sum_{i=1}^{N_{\text{on}}} \gamma_q^i < \gamma_q^{\text{ref}}$ **do**
 $N_{\text{on}} = N_{\text{on}} + 1$
 if $N_{\text{high}} < \frac{2^{b_{\text{ref}} - b_{\text{low}}} - 1}{2^{b_{\text{high}} - b_{\text{low}}} - 1} N_r$ **then**
 $N_{\text{high}} = N_{\text{high}} + 1$
 $b_i = b_{\text{high}}$
 else
 $b_i = b_{\text{low}}$
 end if
end while

Carlo simulation with results averaged over 1000 independent realizations. We discuss the performance for different combinations of b_{low} and b_{high} for both algorithms. In the simulations, the transmitter is always equipped with 4 antennas whereas the number of receive antennas can be either 64 or 256. We consider a mmWave link with $B = 1$ GHz and vary the unquantized link SNR from -20 to 20 dB (except for Figures 3(a) and 3(b), where it is varied from -20 to 30 dB) with a step of 5 dB.

We display the normalized power consumption ξ (10) vs the unquantized SNR for systems that achieve the same quantized SNR (and thus, achievable rate). We use references that have 5 or 4 bits resolution, which achieve the best energy efficiency (EE) of a fixed-resolution system according to [19], [20], [23]. Also note that the SNR does not include antenna gain and therefore -20 and 0 dB SNR corresponds to an approximate communication range of 100 m for NLOS and LOS, respectively. Moreover, mmWave communication is expected to be short range, and therefore higher SNR between 0 and 20 dB is reasonable for shorter range LOS scenarios.

A. GBA

We begin with the results for the GBA algorithm. Figs. 3(a) and 3(b) show the normalized power consumed by the receivers with GBA for $N_r = 64$ and $N_r = 256$ receive antennas, respectively, and $b_{\text{ref}} = 5$. The result shows that above certain SNR values the variable resolution architecture displays lower power consumption than the fixed resolution architecture.

Note that the configurations where $b_{\text{low}}, b_{\text{high}}$ are relatively close to b_{ref} (e.g., $(b_{\text{low}}, b_{\text{high}}) = (4, 6)$) result in a reduced power consumption for almost the complete range of unquantized SNR. This is due to the fact that, in circumstances where the quantization noise of the reference is smaller or comparable to the unquantized signal noise, using a higher b_{low} (i.e., close to b_{ref}) already achieves an achievable rate close to the reference, and thus it only takes very few RF chains with b_{high} bits to close the gap.

Secondly, note that in the settings with very low b_{low} (i.e., with $b_{\text{low}} = 1, 2$ bits) a large power is saved at very high SNR, but the variable resolution system consumes even more power than the reference at low SNRs. This is due to the fact that if the number of bits is sufficient,

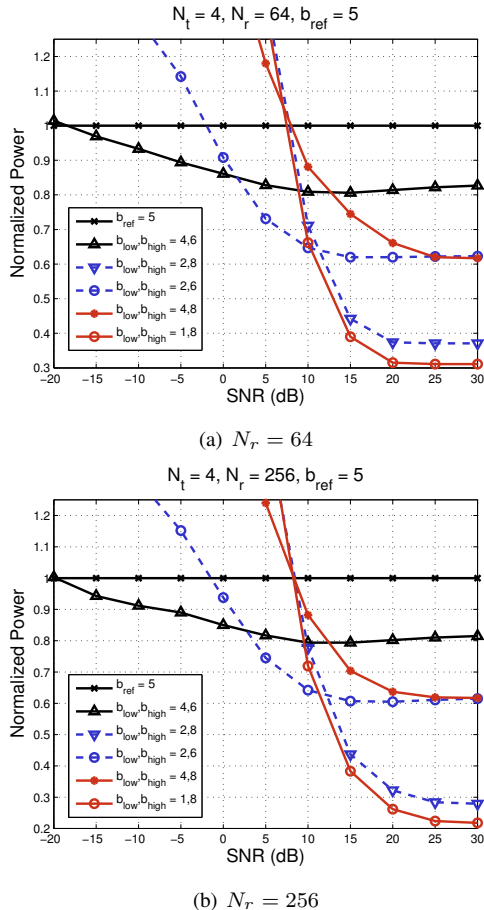


Figure 3. Greedy Bit Allocation with $b_{\text{ref}} = 5$

a quantized system operates very close to the achievable rate of an unquantized system, but the threshold that marks this “sufficient” number of bits grows with the unquantized SNR [8]. Therefore, at high operating SNR the contribution of RF chains with b_{low} is not significant, and the use of smaller b_{low} saves power without harming the achievable rate much, while on the other hand the few ADCs with very high resolution (for instance $b_{\text{high}} = 8$) improve the SNR much more than many RF chains with moderate resolution. Therefore the combination of smaller b_{low} and higher b_{high} works better at high SNR.

Conversely, at low SNR, to achieve the achievable rate of the reference resolution system, a dramatic increase in the number of high resolution ADCs is required, which increases the power consumption of the variable bits scheme even more than the fixed bits reference. For instance, with $(b_{\text{low}}, b_{\text{high}})$ set to $(1, 8)$, $(2, 8)$ and $(4, 8)$, the consumed power is lower than the reference only when the SNR is above 10 dB. This is because b_{high} produces no significant improvement in the SNR and choosing b_{high} closer to b_{ref} results in a lower power consumption.

The impact of the number of receive antennas can be observed by comparing Figs. 3(a) and 3(b). For instance, with $(b_{\text{low}}, b_{\text{high}}) = (4, 6)$, the power of the variable resolution system compared to the fixed resolution reference is in the range 80-95% for both antenna configurations $N_r = 64$ and $N_r = 256$. On the other hand, at the same high SNR of 20

dB, the variable architecture with $(b_{\text{low}}, b_{\text{high}}) = (1, 8)$ can achieve a power saving of 69% with 64 antennas, whereas the system with 256 antennas can achieve a power saving of 75%. It seems that the number of antennas affects the gains more for more extremely separated resolution values that work well at high SNR, whereas it has a negligible impact for narrowly separated resolution values that work well at all SNRs.

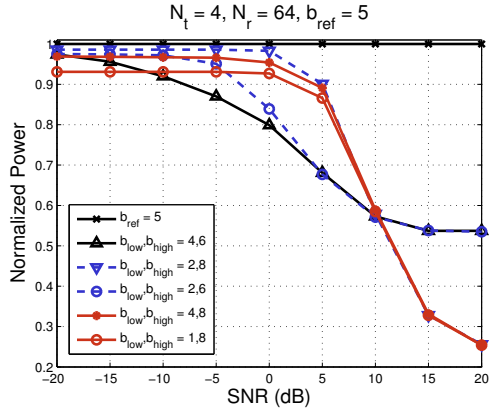
Finally, note that there is a certain operating SNR after which the normalized power saturates. This is due to the fact that at very high SNR, few high bits ADCs are enough to get the same achievable rate as a receiver where all antennas are connected to fewer bits ADCs (i.e., with $b_{\text{ref}} = 5$). Moreover, with $(b_{\text{low}}, b_{\text{high}}) = (4, 6)$, the minimum power is achieved at 10 dB and 15 dB SNR for $N_r = 64$ and $N_r = 256$, respectively. This is due to the phenomenon explained above that at very low (high) unquantized SNR the contribution of b_{high} (b_{low}) to the quantized SNR is not very significant.

B. GASBA

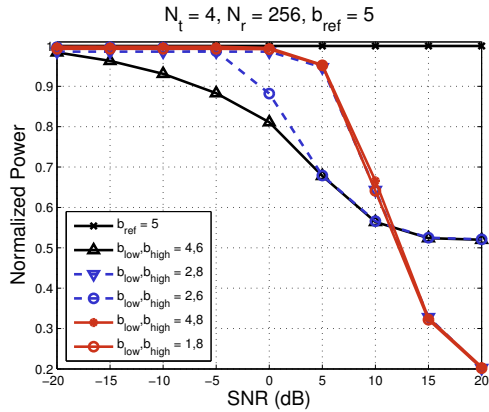
To address the shortcomings of GBA, specially at low SNR, we now discuss the performance of GASBA which also puts constraints on the total power consumed by the high-resolution antennas. Note that GBA always tries to achieve the rate of the reference, and sometimes this causes the power consumption to be greater than the reference. However, in GASBA, the maximum N_{high} (11) ensures that the power consumption of the variable architecture is bounded by the reference.

Figs. 4(a), 4(b) show the results for GASBA with $b_{\text{ref}} = 5$, $N_r = 64$ and $N_r = 256$, respectively. Note that the normalized power of any variable resolution configuration at any operating SNR is less than or equal to the power consumed by the reference architecture, thanks to the design of the algorithm that stops activating high-resolution RF chains when a certain limit is reached. In Fig. 5 we can observe the price to pay for this, in the form of a reduction of rate compared to the reference. Although some schemes have a large penalty, we can see in Fig. 5 that if we pick b_{low} and b_{high} carefully the rate drop can be less than 2%.

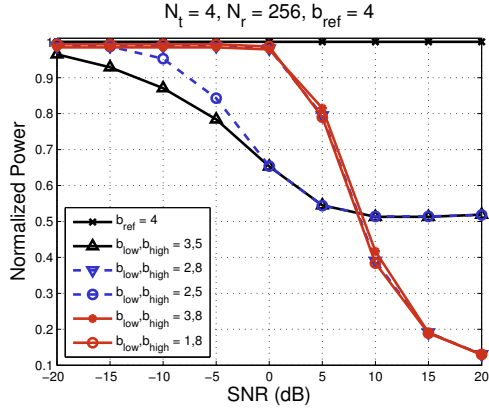
At the lowest SNR values, the normalized power consumption of GASBA becomes flat as all antennas are assigned a non-zero number of bits. The best antennas are assigned b_{high} bits, up to the maximum value of N_{high} in Eq. (11), and the rest of the antennas are all assigned to b_{low} -bit ADCs. In these scenarios, if the margin between b_{low} and b_{high} is narrow, the GASBA algorithm performs like GBA with parameters b_{low} and b_{high} . On the other hand, if $(b_{\text{low}}, b_{\text{high}})$ is wide, the GASBA algorithm gives up trying to match the rate of the reference system in order to avoid the excess power consumption seen in Figs. 3(a) and 3(b). Also note that, for some configurations of $(b_{\text{low}}, b_{\text{high}})$, the value of N_{high} obtained by solving Eq. (11) with equality may not be an integer, and therefore in those cases the floor of N_{high} is selected. Due to this reason the curves for some configurations of $b_{\text{low}}, b_{\text{high}}$ stay below the reference even when all antennas are utilized (Figure 4).



(a) $b_{\text{ref}} = 5$, $N_r = 64$



(b) $b_{\text{ref}} = 5$, $N_r = 256$



(c) $b_{\text{ref}} = 4$, $N_r = 256$

Figure 4. Greedy Antenna Selection and Bit Allocation

At high SNR, power savings with GASBA improve in two ways. As we observed in GBA, choosing a wide $(b_{\text{low}}, b_{\text{high}})$ can lead to a very high gain. In addition, at higher SNRs using only some RF chains with very high b_{high} bits is sufficient to achieve the same rate as the reference. This makes GASBA in the high-SNR regime behave similarly to a GBA algorithm with parameters $(0, b_{\text{high}})$, and therefore the power saving of GASBA at high SNR is the same regardless of b_{low} . This means that GASBA can select b_{low} to work well at low SNR without penalizing its high SNR performance.

Therefore, our results show that the GASBA algorithm

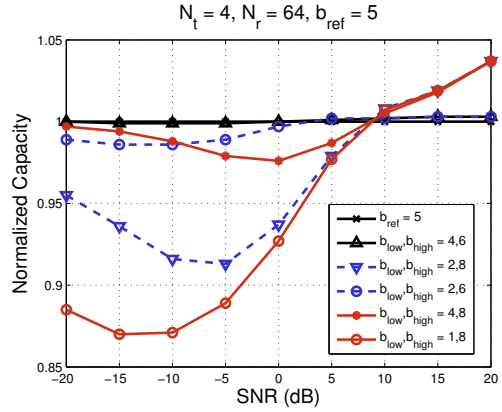


Figure 5. GASBA does not always match the reference achievable rate at low SNR, but careful parameter selection keeps the margin under 2%.

performs quite well for all the SNR range if we select the values of b_{low} relatively close to b_{ref} , and/or pick b_{high} relatively close to b_{ref} (i.e., $(b_{\text{low}}, b_{\text{high}}) = (4, 6)$, $(2, 6)$ or $(4, 8)$). For values selected in this way, the GASBA algorithm achieves at least 98% of the reference rate, a consistent small power saving at the low SNR, and large power savings in the high SNR regimes.

Conversely, if we select very low values for b_{low} and a wide margin from b_{high} , the GASBA algorithm performs poorly at low SNR. Therefore, at low SNR, GBA consumes more power for the same rate, whereas GASBA results in a slight rate loss for maintaining the same power as the reference. As in GBA, at higher SNRs, the systems with very high b_{high} start to display the best power savings. However, b_{high} must not be too extreme for moderate SNR values. For example, at 10 dB the configuration $(2, 6)$ actually performs better than $(2, 8)$. Nonetheless, the curves for $b_{\text{high}} = 8$ display a more abrupt decrease, and the value of b_{low} is irrelevant at high SNR because those RF chains are never turned on.

Note that in the entire SNR range -20 to 20 dB, which is quite reasonable for wireless communication, the variable resolution scheme with $(b_{\text{low}}, b_{\text{high}}) = (4, 6)$ always results in a reasonable power reduction without any degradation in the achievable rate for both GASBA and GBA algorithms. Thus, our results provide an example where the extreme two-value variable-resolution model in [15], with $b_{\text{low}} = 1$, may not be the best configurations for mmWave systems.

Finally, in Fig. 4(c) we observe the effect of changing the reference for a variable resolution GASBA system compared versus $b_{\text{ref}} = 4$. The results show that with a reduction in b_{ref} the difference in power consumption between the fixed and variable resolution techniques increases (compare Figs. 4(b) and 4(c)). This is because a reduction in b_{ref} also reduces the target reference achievable rate and therefore the required number of ADCs with b_{high} bits decreases.

In summary, the use of variable resolution ADCs can provide a significant decrease in the power consumption in comparison to a fixed resolution ADC architecture, ranging from a robust 10-20% for SNR between -5 dB and 30 dB (GBA Algorithm), using narrow $(b_{\text{low}}, b_{\text{high}})$ differences, up

to 50% at 20 dB SNR (GASBA Algorithm).

V. CONCLUSIONS

In this paper, we proposed variable resolution quantization in fully digital receiver architectures with large antenna arrays. This variable-resolution ADC approach can be seen as a generalization of antenna selection and 1-bit ADC selection proposals. It may also be regarded as a practical implementation constraint that simplifies other full range bit allocation optimization models proposed in the literature.

We discussed a model for a mmWave uplink scenario where the transmitter exploits the sparsity of the channel sending a scalar signal projected over the single dominant eigenvector of the channel matrix. We have noted the usual power-consumption model for ADCs and proposed an architecture abstraction representing the behavior of two-level variable resolution ADC components. We have also designed two algorithms to operate within our model, one that merely alternates between high and low resolution levels for the ADCs, and one that adds a third zero-bit resolution level equivalent to antenna-selection techniques.

We have shown that there can be very significant power savings up to 80% in the high SNR regime using variable-resolution quantization schemes with a wide margin between the low and high resolution levels. Unfortunately, these gains are lost at low SNR.

Conversely, selecting pairs of low and high resolution options with not-so-low and not-so-high values can produce very robust gains for any received SNR, but in this case the power saved is only in the order of 10% or so.

Finally, the algorithm that incorporates antenna selection mechanisms can benefit from both trends by selecting between allocating high resolution, moderate resolution, or switching off each antenna separately. This allows to obtain some mild gains in the low SNR regime and some significant gains in the high SNR regime.

Provided that the number of bits is chosen appropriately for a given SNR, either approach outperforms the existing literature proposal that considers 2-level variable resolution systems but always assigns 1 bit to the ADCs with lower resolution.

In the future, we will extend the analysis of variable resolution ADCs to transmissions with spatial multiplexing in more than one dimension, explore schemes with a higher number of resolution levels to choose from and optimize the choice of the variable bits based on the power and achievable rate constraints, and study the relationship between practical constraints in our model and full-range bit allocation optimization algorithms.

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