Can We Improve Over Weber Sampling of Haptic Signals?

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***Abstract*— In applications such as telesurgery, it is required to transmit haptic signals to a remote location with a delay of at most few milliseconds. To reduce the packet rate and yet retain perceptual quality, adaptive sampling has been explored in the literature. In particular, in earlier work we proposed and analyzed an adaptive sampling scheme based on Weber's law of perception. In this paper, we explore other possible adaptive sampling candidates. We describe an experimental setup where users are subjected to piecewise constant haptic stimuli to which they can respond with a click. We record the clicks and ask the question: can we identify signal features and classiers to predict the clicks? The answer suggests adaptive sampling schemes that improve over Weber sampling.**

I. Introduction

As devices for sensing and rendering of haptic signals proliferate, it is natural to ask if haptic signals can be effectively communicated over an existing communication network such as the internet. In order to maintain stability and good quality of perception, it is common in closed loop systems - such as the teleoperation system ([4], [6], [19], [23]) - to sample haptic signals in excess of 1 KHz. To avoid delays, only a few samples can be encapsulated into a data packet, and this leads to a high packet generation rate, which is not desirable. Thus the question arises whether we can use adaptive sampling (that is, sampling that depends on the signal) to transmit only perceptually significant portions of the haptic signal and reduce the average packet rate? This paper aims to develop insight into good structures for adaptive sampling of haptic signals.

In the recent past, several authors have attempted the compression of haptic signals. For example, [27] uses adaptive sampling along with differential pulse code modulation (DPCM) to compress haptic signals, [31] exploits the sparsity of the discrete cosine transform (DCT), and [25] uses predictive coding based on the least squares method and median filtering. These methods process blocks of data and introduce a processing delay, which is not suited for real time applications. For real time applications, several authors have attempted to exploit Weber's law of perception to sample the haptic signal - see for example [10], [11], [15], [16], [17], [18], [20], [26], [29], [32], [33]. Weber's law postulates that perception depends on percentage change in the signal with respect to a reference, and hence if Weber's law is true, then we only need to sample at points where the percentage change is high. This main idea is exploited for adaptive sampling of haptic signals in [15], [18]. In [10], the deadband behavior of the Weber's law is used to reduce the impact of delay in teleoperation. In [16] , multi-dimensional haptic data is considered. A comparison of fixed rate sampling and adaptive sampling based on Weber's law is given [33]. In [11], a Weber sampler motivated by these other works is defined and analyzed in detail. In particular, [11] provides expressions for the sampling rate and inter-sample time of the Weber sampler for a wide class of smooth signals.

While Weber's law is well studied, the exact nature of haptic perception is not fully understood (see for example [8], [13], [24]). A basic question is whether in a typical environment some other sampling strategies work as well as the Weber sampler or even better? In this paper, we present evidence that some other simple adaptive strategies may be as good or better than those based on Weber's law. We describe a response prediction experiment where we use a Phantom Omni haptic device [2], [28] along with HAPI [1], [21] to subject users to a haptic force. The force is generated to be piecewise constant and the instants of jump are clearly identified as the only points that are perceptually significant. We ask the user to click a stylus whenever he/she feels a perceptible change in the force. We record the clicks of the user for a large number of signals. After accounting for the response time of the user, we can label each jump in the haptic signal as "perceived" (label 1) or "not perceived" (label -1). Using this labeled data, our aim is to build classifiers that use suitable features of the signals and predict the labels of the jumps in the signal. Our thesis behind this approach is that a classifier with high accuracy captures the perceptually important structure in the signal and can also be used for sampling the signal. Since we are interested in causal adaptive samplers, we restrict our attention to classifiers based on causal features. Specifically, we use classifiers based on Weber's law, level crossings, and linear regression. The first two classifiers depend only on the signal value at the latest two jumps and we show evidence that incorporating even further past samples improves accuracy but only marginally. We find that the level crossings based classifier has a slight edge over the Weber's law based classifier, but the gain in accuracy is within a standard deviation (computed based on 40 runs of hold-out cross-validation). The Weber and level

Fig. 1: Experimental set up, user holding the device to feel the force

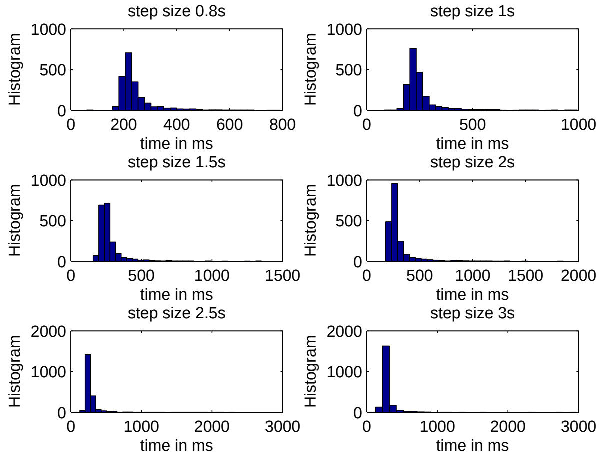

Fig. 2: Histograms of response time for stair-case signals with different time spacing.

crossings classifiers have about 93% accuracy and there are natural adaptive samplers based on these classifiers. These classifiers are based on the latest two jumps of the signal. We also propose a classifier based on linear regression of the latest three jumps in the signals and we find that it attains an accuracy of about 95%. Thus the addition of further past samples helps, but the impact is limited.

We note that the classifiers we have identified are not the only ones and even more sophisticated classifiers can be employed. (We hope to report more such results in a subsequent publication.) Our main point is that we can take a completely data driven approach to synthesizing good candidates for adaptive samplers: we can build classifiers that work well on the experimental data and each such classifier gives us a potential adaptive sampler. In particular, we have identified two classifiers which perform better than the Weber classifier and hence adaptive samplers corresponding to them are also of interest. Adaptive samplers designed with this approach can then be tested by more experiments to study their compression-distortion tradeoff, but this is beyond the scope of this paper.

We also note that our aim is *not* to study laws of perception. Our focus in on classification of perceptually significant points in the haptic signals in a *realistic* environment. Hence we do not make any special effort to isolate the user from any ambient disturbances. Our data is collected over several weeks with varying ambient conditions and yet we get good classification accuracy. This further underscores the utility of the classifiers (and their associated features) in realistic environments.

The paper is organized as follows. In Section II, we describe the experimental setup and labeling of the data. In Section III, we describe the parameter learning for the classifiers, and compare the accuracy of the classifiers. The conclusion is given in Section IV.

II. Experimental Set Up

In this section, we describe the our experimental setup and data collection process.

A. The Haptic Device

We use a Phantom Omni [2], [28] haptic device along with HAPI [1], [21], an open source software platform, to calculate and send a kinesthetic haptic force to the user. A fire wire port is used to communicate between a computer and the haptic device. The relevant specifications of the haptic device are as follows.

1) Maximum force : 3.3*N*
2) Force feedback workspace : Width 160mm, Height 120mm, Depth 70mm
3) Force update frequency: maximum of 1 KHz, that is, once every msec.

The haptic device has a detachable stylus, which can be held like a pen as shown in Figure 1. The stylus has six degrees of freedom, but we only consider 1-D haptic force in this paper. The stylus has two programmable buttons and one button is used to record the response of the user, who feels the haptic force by holding the stylus and presses the button on the stylus if he/she perceives a change in the force.

B. Signal

We use piecewise constant signals since the jumps in such signal are clearly the only points where the perception can change. This allows us to associate user response with specific points in the signal. The signals we generate have a parameter T_0 - the time separation between the jumps. For a given signal, T_0 is fixed and we consider T_0 in the range of 0.8 to 3.0 seconds. Thus the signal changes values only at the time instants $T_0, 2T_0, 3T_0, ...$ and is constant in between these time instants.

The value of the signal at time nT_0 is generated independently of all previous values and is generated with a uniform distribution over the range $[0, 3]$. This ensures that we cover almost the entire force range of the haptic device. If the signals have a pattern, such as an increasing or decreasing staircase, then the human mind can potentially anticipate such patterns. Hence we have used random signal levels, which ensure that

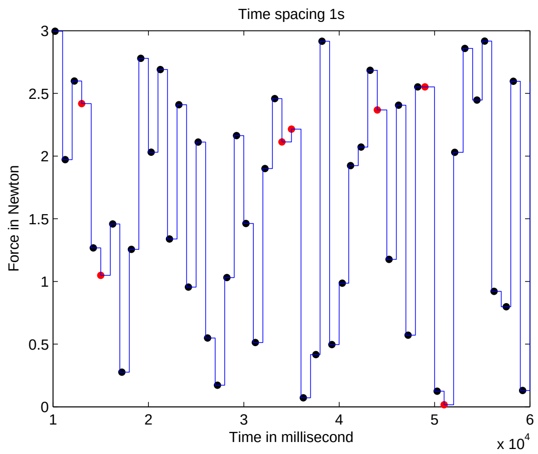

Fig. 3: A typical realization of the force signal and its labeling. The black dots represent jumps that are perceived. The red dots represent jumps that are not perceived.

there are no specific patterns in the signal that can bias the perception of the signal.

C. *Recording Response and Labeling Jumps*

As explained above, the user is subjected to a 1-D kinesthetic force signal and is asked to press the button of the haptic device whenever he/she feels a change in the force. The human response has a non-zero delay and also each button press is not instantaneous but lasts for a few milliseconds. We need to account for these factors in our experiments and T_0 cannot be too small. To determine the response time, we generated 25 runs of an increasing staircase signal for different spacing between the steps. After each jump in the signal, we record the time instants (with a resolution of 1 msec) when the button is pressed. In Figure 2, we plot the histograms of the response time. We see that it varies between 200 to 500 msec. Hence in all our experiments, we have chosen $T_0 \geq 0.8$ seconds. We also note that as the spacing between the jumps in the staircase increases, the response concentrates more (roughly around 300 msecs).

Once we have ensured that T_0 is large enough so that the response to a jump does not spill over to the next interval, it is easy to label the jumps of the signal. We say that a user has perceived a jump if we record a click from the user within the interval of length T_0 following the signal jump. Otherwise the signal jump is labeled as not perceived. A typical realization of the signal and the corresponding labels are illustrated in Figure 3.

D. *Data Statistics*

In this paper, due to space constraints, we report results for one user; we hope to report results for more number of users in subsequent publications. The user's sole task is to feel the force and give his/her feedback by clicking the button on the stylus. We note that our goal is not to propose laws of perception, but merely to classify jump points as perceived or not in a *realistic* environment. Hence we have not made any special efforts to screen the user from other distractions, but neither have we subjected the user to any explicit distraction. The data has been collected over about four weeks and thus spans a variety of ambient conditions. Each signal is chosen to have 100 jumps and hence it is of duration $100T_0$. Since T_0 varies from 0.8 to 3 seconds, the signal duration varies from 80 seconds to 300 seconds. For each T_0, we subject the user to 25 independent runs over a period of few hours. Thus for each time spacing T_0, we have 2500 labeled jumps. For $T_0 = 1$ second, the fraction of perceived jumps is about 85%.

III. Classification of Jumps in the Signal

Our aim is to study choice of features and classifiers which predict the label based on these features. In Section III-B, we study a feature and a classifier suggested by Weber's law. In Section III-C, we study a feature and classifier based on level crossings, and also show that additional improvement is possible using classifiers based on further past samples and linear regression. But first we state the method of performance evaluation.

A. *Performance Evaluation Methodology*

We consider a number of different classifiers. If $\boldsymbol{X}_n$ denotes the feature vector used by the classifier, $Y_n \in \{-1, 1\}$ is the true label, and $h(\cdot)$ is the classifier, then the error rate of the classifier is

$$E_H = \frac{1}{N} \sum_{i=1}^{N} 1(h(\boldsymbol{X}_i) \neq Y_i). \tag{1}$$

The classifier may have parameters that we would like to optimize and we also use other alternate expressions for the error rate in subsequent sections.

For training of the classifier and evaluating its performance, we use holdout cross-validation [22]. Consider the collection of all jumps in the different runs for a fixed T_0. We randomly split the set of jumps into two equal parts such that each part has the same proportion of labels as the original data, that is, we use *stratified sampling*. One part is used for training and the other for testing. To ensure that the results are not biased by a specific partitioning of the data, we repeat this procedure independently 40 times and report the error rate of the classifiers averaged over the 40 realizations.

B. *Weber Classifier*

The Weber's law states that perception depends on percentage changes in signals and in addition to haptics it has been reported for a variety of other perceptual signals such as vision, audio, smell (see for example [5], [12], [14], [30]). At the n^{th} jump of the signal, let X_n denote the signal value, and let X_{n-1} be the value before the jump. Then Weber's law suggests that the jump is perceived if and only if

$$\left| \frac{X_n - X_{n-1}}{X_{n-1}} \right| \geq \delta. \tag{2}$$

where $\delta > 0$ is the Weber constant. We call this as the Weber classifier and we minimize its error over δ using the training

Pulse duration in seconds	E_w	σ of E_w	δ_{opt}	σ of δ_{opt}
0.8	0.07666	0.00475	0.12944	0.00479
1.0	0.06342	0.00500	0.12674	0.00399
1.5	0.08416	0.00559	0.12628	0.00535
2.0	0.07411	0.00507	0.13665	0.00456
2.5	0.06345	0.00534	0.11654	0.00460
3.0	0.06220	0.00534	0.11655	0.00429

TABLE I: Weber classifier

Pulse duration in seconds	E_l	σ of E_l	c_{opt}	σ of c_{opt}
0.8	0.07467	0.00519	0.23608	0.00654
1.0	0.06008	0.00543	0.24183	0.00551
1.5	0.07821	0.00562	0.24200	0.00614
2.0	0.07524	0.00617	0.25775	0.00740
2.5	0.06015	0.00619	0.20462	0.00649
3.0	0.05815	0.00472	0.21413	0.00538

TABLE II: Level crossing classifier

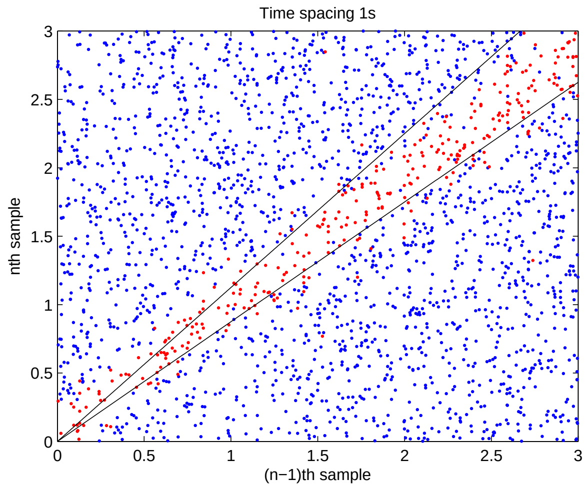

Fig. 4: Scatter plot of the stair case signal with time spacing 1s. n^{th} sample is present sample and $(n-1)^{th}$ sample is the previous perceived point. Blue and red points represent perceived and not perceived points respectively with respect to previous perceived point. Black lines are the Weber boundaries as suggested by Weber classifier. Slopes of these boundaries are determined by Weber constant δ.

set. Let R be the total number of runs and let I_r be the jumps in run r that are part of the training set. Then, for the optimization of the parameter, it is convenient to express the error rate for this classifier in the following form:

$$E_w(\delta) = \frac{1}{4N}\sum_{r=1}^{R}\sum_{i\in I_r}(Y_i - \text{sign}((X_i - X_{i-1})^2 - (\delta X_{i-1})^2))^2. \quad (3)$$

We note that if the classifier is correct, then the summand is zero, but otherwise it takes the value 4, and hence we have a factor of $1/4$ outside the sum. Based on a plot of the error rate as a function of δ we believe that there is a single global minimum and the gradient descent algorithm can find this minimum. Since sign(x) is discontinuous, for the sake of implementing the gradient descent algorithm, we replace it by the hyperbolic tangent function $\tanh(10x)$. Once the optimal parameter is learnt for the training set, we apply the classifier with the optimal parameter to the test dataset.

In Table I we summarize the results for the Weber classifier. We see the average error rate across 40 holdout realization is quite small - in the range of 6-8%. The standard deviation is an order of magnitude smaller, indicating that the error rate estimate is quite good. The optimal value of δ (averaged over the 40 holdout realizations) varies from 11.6% to 13.6%, which is in the same range as studies of the Weber constant in prior literature (see for example [3], [7]). There does not appear to be any specific relationship between T_0 and δ_{opt}, but for largest two values of T_0 considered, δ_{opt} is smallest.

In Figure 4, we illustrate the Weber classifier for the case of $T_0 = 1$ second. We see that the classification errors are primarily for very small or very large amplitudes. In the next section, we see that we can improve over the Weber classifier.

C. Classification Based on Level Crossings and Linear Regression

Instead of looking at percentage change as in the case of the Weber's classifier, we could look at absolute difference: Classify as 1 if $|X_n - X_{n-1}| > c,$ else classify as -1. We call this the level crossings classifier. The error rate of this classifier depends on the parameter c and we can write it in the form

$$E_l(c) = \frac{1}{4N}\sum_{r=1}^{R}\sum_{i\in I_r}\left[Y_i - \text{sign}\big((X_i - X_{i-1})^2 - c^2\big)\right]^2 \quad (4)$$

where the summation is over all samples in the training set. To find the optimal c, we once again replace sign(x) by $\tanh(10x)$ and use the gradient descent algorithm. The use of gradient descent is based on our observation that a plot of the error rate with respect to c reveals a single global minimum. The classifier is applied to the test set using the optimal parameter value found on the training set.

In Table II, we show the average and variance of the error rate and the optimal value of c computed over 40 realizations of the holdout. We see that the level crossings is quite good with an error rate in the range of 6-8 %. It is consistently better than the Weber classifier, but the gain is within one standard deviation of the error rate. The optimal value of c varies from 0.2 to 0.25 N and there does not appear to be any specific relation between the optimal value and T_0. In Figure 5, we illustrate the level crossings classifier for the case of $T_0 = 1$ second.

The success of the level crossings classifier raises a natural question: can a more complex linear regressions improve performance further? To answer this question, we consider a

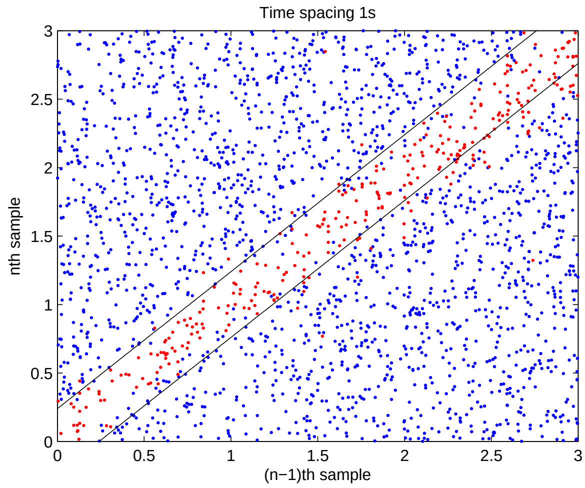

Fig. 5: Scatter plot of the stair case signal with time spacing 1s. n^{th} sample is present sample and $(n-1)^{th}$ sample is the previous perceived point. Blue and red points represent perceived and not perceived points respectively with respect to previous perceived point. Black lines are the level crossings boundaries as suggested by level crossings classifier. Intercept of these lines on the axes are determined by constant c.

classifier that declares 1 if

$$|a_0 X_n + a_1 X_{n-1} + a_2 X_{n-2}| \geq 1 \tag{5}$$

and declares a -1 otherwise, where $a_0 > 0$, a_1 and a_2 are real valued constants. The level crossings is a special case with $a_0 = -a_1 = 1/c$ and $a_2 = 0$. The error rate can be expressed as

$$E_g(a_0, a_1, a_2) = \frac{1}{4N} \sum_{r=1}^{R} \sum_{i \in I_r} \left[Y_i - \text{sign}\left((a_0 X_i + a_1 X_{i-1} + a_2 X_{i-2})^2 - 1\right)\right]^2 \tag{6}$$

The error rate depends on the three parameters and in general it appears to have several local minima. Hence, we cannot use the gradient descent algorithm. To ensure that we do not get trapped in a local minima, we use the simulated annealing algorithm (see for example [9]). The simulated annealing algorithm is initialized with a random state. The neighbors of the current state are selected from a Gaussian distribution with the current state as the mean and a standard deviation of 0.5. The algorithm has a temperature parameter, which is initialized to 50. For a given temperature, we run 5000 iterations of simulated annealing. Then we reduce the temperature by 10% and continue the iterations. The algorithm is stopped when the temperature falls below 0.01 or the number of steps exceeds 100,000. With these parameters, it takes more than 20 hours to compute the optimal parameter values for our datasets.

In Table III, we show the parameter values and error rate of the linear regression based estimator averaged over 40 realizations of holdout. We see a clear improvement over the level crossings classifier, and except for the case of $T_0 = 1.5$ second, we see that the gain in accuracy is about 2%. Since the accuracy of level crossings is already around 93%, this additional increase, is small. We see that a_0 and a_1 have similar magnitudes and opposite sign for all values of T_0. Also a_2 has a much smaller magnitude than a_0, a_1. The level crossings classifier has parameters with the same behavior ($a_0 = -a_1$ and $a_2 = 0$) and it is not surprising that it does well. For larger values of T_0, a_2 takes smaller values, that is, the importance of X_{i-2} diminishes. Thus for larger T_0, there is a higher tendency to 'forget' the more distant past X_{i-2}.

Pulse duration in seconds	E_g	a_2	a_1	a_0
0.8	0.05178	-0.27078	-4.47695	4.81312
1.0	0.04553	-0.22298	-5.52813	5.85001
1.5	0.07630	-0.27793	-4.79109	5.10514
2.0	0.06211	-0.08375	-5.11101	5.24511
2.5	0.04159	-0.14616	-4.79609	5.06772
3.0	0.04673	-0.13988	-5.13003	5.34953

TABLE III: Linear Regression Based Classifier

IV. Conclusion

In order to identify good adaptive sampling strategies for haptic signals, in this paper we record the response of a user to several haptic signals and classify the perceptually significant points as perceived or not perceived. Our thesis is that classifiers that work well should be considered as candidates for building adaptive sampling strategies. Our results show that we can improve over the Weber classifier, whose corresponding sampler - the Weber sampler - has been studied by many authors. The level crossings classifier is marginally (but consistently) better than the Weber classifier, and about 2% further improvement is possible by considering further past samples and a linear regression based classifier. However, since the Weber classifier itself has good accuracy - in excess of 92% - the additional gain has limited value. The ideas of level crossings and linear regression can be easily incorporated in adaptive sampling strategies. Based on our results, we think that there are a number of simple classifiers and associated sampling strategies that may perform as well or better than those based on Weber's law. In this sense, the import of Weber's law for sampling of haptic signals in a realistic environment is limited, even though its performance is good. More analysis of implementation complexity and rate-distortion tradeoff needs to carried out to understand which sampling mechanisms are most suited in applications. In addition, it is possible to consider more sophisticated classifiers, which have the potential to be better. In future work, we hope to pursue such data analysis including data from several users.

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