On the Duality between Multiple-Access Codes and Computation Codes

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***Abstract*—For a two-user Gaussian multiple access channel, computation codes are designed for the scenarios where the decoder is *not* interested in decoding the two codewords, but only the sum of them. It has been observed that good computation codes should possess some algebraic structure. In this note, we expose the fact that such algebraic structure could undermine the capability of the codes for recovering the messages, *i.e.*, for the purpose of multiple-access. Particularly, we establish duality results between the codes which are good for computation and the codes which are good for multiple access.**

I. Introduction

In communication systems, conventional coding schemes use error correction codes to combat noisy channels, and require receivers to decode the transmitted codewords, hence recover the corresponding messages reliably. More recently, many advanced coding techniques proposed for multi-user communication systems require the receiver to decode a function of codewords from multiple transmitters, rather than recover the individual messages. As a canonical example of such, consider the scenario where the receiver in a two-user multiple access channel is asked to decode the sum of the two transmitted codewords. This *computation* aspect of these schemes is important, sometimes even imperative in multi-user communication networks. Results from network coding [1] [2], physical network coding [3], and the compute-and-forward scheme [4] have all shown that computing certain function of codewords within a communication network is vital to the overall coding strategy, and their performance cannot be achieved otherwise.

It is well known that the codes with algebraic structure are preferred when we design decoding schemes for computation. For example the nested lattice codes are used in the compute-and-forward [4]. On the other hand, some recent work (e.g. [5]) suggests that such algebraic structure could be obstacles for the purpose of multiple-access. This note makes a simple yet interesting observation on the duality between the codes used for communication and the codes used for computation on the two-user Gaussian multiple-access channel (MAC).

Throughout the note, we will use $[n]$ to denote the set of integers $\{1, 2, \ldots, n\}$ for some $n \in \mathbb{Z}_+$.

II. Problem Statement and Definitions

To start with a formal definition of communication codes and computation codes for the MAC, consider the symmetric two-user Gaussian MAC

$$Y^n = x_1^n + x_2^n + Z^n \tag{1}$$

where $Z_i \sim \mathcal{N}(0, N)$, $i \in [n]$ is the additive white Gaussian noise and both channel inputs have the same average power constraint $\sum_{i=1}^n x_{ki}^2 \leq nP$, $k = 1, 2$. For the sake of notation, we will define the signal-to-noise ratio (SNR) to be $\mathsf{SNR} := P/N$, and denote such symmetric two-user Gaussian MAC as GMAC(SNR).

Definition 1 (Multiple-access codes): A $(2^{nR_1}, 2^{nR_2}, n)$ multiple-access code[1] for GMAC(SNR), consists of

- two message sets $[2^{nR_k}]$, $k = 1, 2$,
- two encoders, where each encoder maps each message $m_k \in [2^{nR_k}]$ to a sequence $x_k^n(m_k) \in \mathbb{R}^n$ *bijectively*, such that $\sum_{i=1}^n x_{ki}^2 \leq nP$,
- a decoder that maps an estimated pair $(\hat{x}_1^n, \hat{x}_2^n)$ to each received sequence y^n.

Each message M_k, $k = 1, 2$, is assumed to be chosen independently and uniformly from $[2^{nR_k}]$. The average probability of error for multiple-access is defined as

$$P_\epsilon^{(n)} = \mathsf{P}\{(X_1^n, X_2^n) \neq (\hat{X}_1^n, \hat{X}_2^n)\}. \tag{2}$$

We say a rate pair (R_1, R_2) is *achievable for multiple access* if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, n)$ multiple-access codes such that $\lim_{n\to\infty} P_\epsilon^{(n)} = 0$.

From the classical capacity results of the Gaussian MAC [6], we know that for the GMAC(SNR), there exists a sequence of $(2^{nR_1}, 2^{nR_2}, n)$ multiple-access codes for any rate pair $(R_1, R_2) \in \mathcal{C}_{mac}(\mathsf{SNR})$ where

$$\begin{aligned}\mathcal{C}_{mac}(\mathsf{SNR}) := \{(R_1, R_2) : &R_1 < \frac{1}{2}\log(1+\mathsf{SNR}) \\ &R_2 < \frac{1}{2}\log(1+\mathsf{SNR}) \\ &R_1 + R_2 < \frac{1}{2}\log(1+2\mathsf{SNR})\}.\end{aligned}$$

The following definition formalizes the concept of computation codes used in this note.

Definition 2 (Computation codes for the Gaussian MAC): A $(2^{nR_1}, 2^{nR_2}, n)$ *computation code*[2] for the GMAC(SNR),

[1] or simply *multiple-access code*, when the parameters are clear from the context.

[2] or simply *computation code*, when the parameters are clear from the context.

consists of two messages sets and two encoders defined as in Definition 1 and

- a decoder that maps an estimated sum $\hat{w}^n \in \mathbb{R}^n$ to each received sequence y^n

As in multiple-access codes, the message M_k, $k = 1, 2$, is assumed to be chosen independently and uniformly from $[2^{nR_k}]$. The average probability of error for computation is defined as

$$P_\epsilon^{(n)} = \mathsf{P}\{X_1^n + X_2^n \neq \hat{W}^n\}. \tag{3}$$

We say a rate pair (R_1, R_2) is *achievable for computation* if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, n)$ computation codes such that $\lim_{n\to\infty} P_\epsilon^{(n)} = 0$.

Since the sum $X_1^n + X_2^n$ can be computed directly at the receiver if the individual codewords (X_1^n, X_2^n) are known, a $(2^{nR_1}, 2^{nR_2}, n)$ multiple-access code for the GMAC(SNR) is also a computation code for the same channel. More interesting are the computation codes with rates outside the GMAC capacity region, i.e. $(R_1, R_2) \notin \mathcal{C}_{mac}(\mathsf{SNR})$. We refer to such codes as *good* computation codes, which is formally defined as follows.

Definition 3 (Good computation codes): Consider a sequence of $(2^{nR_1}, 2^{nR_2}, n)$ computation codes for the GMAC(SNR). We say that the computation codes are *good* if

$$R_1 + R_2 > \frac{1}{2}\log(1 + 2\mathsf{SNR}),$$

namely, the sum rate is larger than the sum capacity of GMAC(SNR).

The compute-and-forward framework have established the existence of good computation codes over the Gaussian MAC where nested lattice codes [7] are used. More recently, nested linear codes are shown to be good computation codes for a general discrete-memoryless MAC [5]. Focusing on the Gaussian MAC, we have the following result [4].

Theorem 1 (Existence of good computation codes): For $\mathsf{SNR} > \frac{3}{2}$, there exists good $(2^{nR_1}, 2^{nR_2}, n)$ computation codes for the GMAC(SNR) such that

$$(R_1, R_2) \in \mathcal{R}_{comp}(\mathsf{SNR}) \backslash \mathcal{C}_{mac}(\mathsf{SNR})$$

where

$$\mathcal{R}_{comp}(\mathsf{SNR}) := \Big\{ (R_1, R_2) : R_1 < \frac{1}{2}\log\left(\frac{1}{2} + \mathsf{SNR}\right), \; R_2 < \frac{1}{2}\log\left(\frac{1}{2} + \mathsf{SNR}\right) \Big\}.$$

III. Duality

In this section we show that for any sequence of code pairs, there is an intrinsic tension between their capability for computation and their capability for multiple-access over GMAC(SNR). Some similar phenomena have already been observed in [5] [8]. Here we make precise statements, showing that over a symmetric Gaussian MAC, a pair of code sequences which are good for computation cannot be used for multiple access, and vice versa.

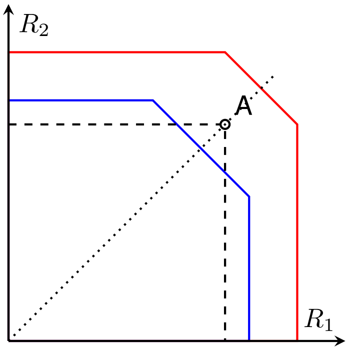

Fig. 1. MAC capacity regions for SNR_1 (blue), SNR_2 (red) where $\mathsf{SNR}_2 >$ SNR1. The point A is achievable by a pair of good computation codes $\mathcal{C}_1, \mathcal{C}_2$ over GMAC(SNR_1) with rate (R_1, R_2). While the rate pair (R_1, R_2) is included in the capacity region of GMAC(SNR_2), the codes $\mathcal{C}_1, \mathcal{C}_2$ cannot be used as multiple-access codes for GMAC(SNR_2).

Theorem 2 (Computation/multiple-access duality 1): Let SNR_1 be a fixed but arbitrary value and let $(\mathcal{C}_1^{(n)}, \mathcal{C}_2^{(n)})$ be a sequence of good computation codes for GMAC(SNR_1). Then this sequence of good computation codes cannot be a sequence of multiple-access codes for GMAC(SNR_2), for *any* SNR_2.

Remark 1: By the definition of good computation codes for GMAC(SNR_1), the rate pair (R_1, R_2) is outside the capacity region $\mathcal{C}_{mac}(\mathsf{SNR}_1)$. Hence it obviously cannot be an achievable rate pair for multiple-access in a Gaussian MAC with an SNR value smaller or equal to SNR_1. The question of interest is if this sequence of good computation codes can be used for multiple-access over a Gaussian MAC when its SNR is much larger than SNR_1. The above theorem shows a fundamental characteristic of such code sequences in that good computation codes cannot be used for multiple access even with an arbitrarily large SNR. Figure 1 gives an illustration of this result.

Proof: To prove the claim, we consider an auxiliary system of the form

$$Y_1^n = x_1^n + x_2^n + Z_1^n \tag{4a}$$
$$Y_2^n = x_1^n + x_2^n + Z_2^n \tag{4b}$$

where the variance of Z_1^n is N_1 per dimension and the variance of Z_2 is N_2 per dimension. Assuming user $k = 1, 2$, is equipped with codebook $\mathcal{C}_k^{(n)} \subseteq \mathbb{R}^n$. The goal of receiver 1 is to recover the sum $X_1^n + X_2^n$ using the channel output Y_1^n, and the goal of receiver 2 is to recover both codewords (X_1^n, X_2^n) using the channel output Y_2^n. In other words, we wish to use one pair of codes as computation codes for receiver 1, and as multiple-access codes for receiver 2 at the same time.

We temporarily assume that the sum $X_1^n + X_2^n$ can be decoded reliably by receiver 1, and the pair (X_1^n, X_2^n) can be decoded reliably by receiver 2. An upper bound on the sum rate $R_1 + R_2$ can be derived as:

$$\begin{aligned} n(R_1+R_2) &= H(X_1^n, X_2^n) \\ &= I(X_1^n, X_2^n; Y_2^n) + H(X_1^n, X_2^n | Y_2^n) \\ &\overset{(a)}{\leq} I(X_1^n, X_2^n; Y_2^n) + n\epsilon_n \\ &= I(X_1^n + X_2^n; Y_2^n) + n\epsilon_n \\ &\leq H(X_1^n + X_2^n) + n\epsilon_n \\ &= I(Y_1^n; X_1^n + X_2^n) + H(X_1^n + X_2^n | Y_1^n) + n\epsilon_n \\ &\overset{(b)}{\leq} I(Y_1^n; X_1^n + X_2^n) + 2n\epsilon_n \\ &\leq \frac{n}{2}\log(1+2P/N_1) + 2n\epsilon_n \end{aligned}$$

where (a) follows from Fano's inequality under the assumption that X_1^n, X_2^n can be decoded at receiver 2 and (b) follows from Fano's inequality under the assumption that $X_1^n + X_2^n$ can be decoded at receiver 1. The last step follows by a standard Gaussian-maximizing-entropy argument. Since $\epsilon_n \to 0$ as $n \to \infty$, we obtain the upper bound

$$R_1 + R_2 \leq \frac{1}{2}\log\left(1 + \frac{2P}{N_1}\right) \tag{5}$$

Under our assumption in the theorem that the sequence of codes $\mathcal{C}_1^{(n)}, \mathcal{C}_2^{(n)}$ are *good* computation codes for the GMAC(P/N_1), we know that the sum $X_1^n + X_2^n$ can be decoded reliably at receiver 1, and furthermore we have the achievable sum rate

$$R_1 + R_2 = \frac{1}{2}\log(1 + 2P/N_1) + \delta \tag{6}$$

for some $\delta > 0$, by Definition 3. However, this implies immediately that decoder 2 *cannot* decode both codewords reliably. Indeed, if decoder 2 could decode both codewords, the achievable sum rate in (6) directly contradicts the upper bound in (5). Notice that this argument holds for any value of N_2, we conclude that the same sequence of codes cannot be used as multiple-access codes for the GMAC(SNR_2), regardless of the value of SNR_2. ■

The following theorem gives a complementary result.

Theorem 3 (Computation/multiple-access duality 2): Let SNR_2 be a fixed but arbitrary value, and let $(\mathcal{C}_1^{(n)}, \mathcal{C}_2^{(n)})$ be a sequence of $(2^{nR_1}, 2^{nR_2}, n)$ multiple-access codes for the GMAC(SNR_2) with arbitrary R_1, R_2. Then this sequence of codes cannot be good computation codes for the GMAC(SNR_1), for *any* SNR_1.

Proof: Consider again the auxiliary system in (4). Under the assumption in the theorem, both codewords (X_1^n, X_2^n) can be recovered at decoder 2 with the rate pair (R_1, R_2). Suppose the sum $X_1^n + X_2^n$ can also be reliably decoded at decoder 1, then it must satisfy

$$R_1 + R_2 \leq \frac{1}{2}\log(1 + 2P/N_1)$$

as shown in the upper bound (5). By Definition 3, this sequence of codes are not *good computation codes* for the Gaussian MAC with decoder 1 as the receiver, since the sum rate is not larger than the sum capacity of GMAC(P/N_1). As the argument holds for any N_1, N_2, we obtain the claimed result. ■

IV. A more general channel model

We point out that the results in the previous section are quite sensitive to the change of channel gains in the system model. It heavily relies on the fact that the coefficients in the sum $X_1 + X_2$ is matched to the unit channel gains. In this section, we extend the results from the previous section to a more general channel model, using the same argument. In this section we use the notation GMAC$(1, a, N)$ to denote the Gaussian MAC of the form

$$Y^n = x_1^n + a x_2^n + Z^n \tag{7}$$

with channel coefficients $(1, a)$ for some $a \in \mathbb{R}$. The white Gaussian noise Z^n has variance N per dimension. Two users are assumed to have the same power constraints P without loss of generality.

The following two theorems are slight generalizations of the results in the previous section.

Theorem 4 (Duality for the general model): Let $(\mathcal{C}_1^{(n)}, \mathcal{C}_2^{(n)})$ be a sequence of good computation codes w. r. t. the sum $X_1^n + a_2 X_2^n$ for the GMAC$(1, a_1, N_1)$. That is, the sum rate of the two codes is larger than the sum capacity of the GMAC$(1, a_1, N_1)$. Then $(\mathcal{C}_1^{(n)}, \mathcal{C}_2^{(n)})$ cannot be a sequence of multiple-access codes for the GMAC$(1, a_2, N_2)$, for *any* choice of a_1, a_2, N_1, N_2.

Remark 2: It should be noticed that the sum to be computed over the GMAC$(1, a_1)$ has coefficients $(1, a_2)$, which is matched to the channel coefficients of GMAC$(1, a_2)$.

Proof: To proceed, we first consider an auxiliary system of the form

$$Y_1^n = x_1^n + a_1 x_2^n + Z_1^n \tag{8a}$$

$$Y_2^n = x_1^n + a_2 x_2^n + Z_2^n \tag{8b}$$

where the variance of Z_1^n is N_1 per dimension and the variance of Z_2^n is N_2 per dimension. The channel gains are $a_1, a_2 \in \mathbb{R}$. In this auxiliary system, the goal of decoder 1 is to recover the sum $X_1^n + a_2 X_2^n$, and decoder 2 wishes to recover (X_1^n, X_2^n). We use GMAC$(1, a_1, N_1)$ and GMAC$(1, a_2, N_2)$ to denote the first and the second Guassian MAC, respectively. If the decoding processes at two receivers are successful, the sum rate is bounded as

$$\begin{aligned}
&n(R_1+R_2)\\
&= H(X_1^n, X_2^n)\\
&= I(X_1^n, X_2^n; Y_2^n) + H(X_1^n, X_2^n | Y_2^n)\\
&\overset{(a)}{\le} I(X_1^n, X_2^n; Y_2^n) + n\epsilon_n\\
&= I(X_1^n + a_2 X_2^n; Y_2^n) + n\epsilon_n\\
&\le H(X_1^n + a_2 X_2^n) + n\epsilon_n\\
&= I(Y_1^n; X_1^n + a_2 X_2^n) + H(X_1^n + a_2 X_2^n | Y_1^n) + n\epsilon_n\\
&\overset{(b)}{\le} I(Y_1^n; X_1^n + a_2 X_2^n) + 2n\epsilon_n\\
&= h(Y_1^n) - h(Y_1^n | X_1^n + a_2 X_2^n) + 2n\epsilon_n\\
&\le h(Y_1^n)\\
&- h(X_1^n + a_1 X_2^n + Z_1^n | X_1^n + a_2 X_2^n, X_1^n + a_1 X_2^n) + 2n\epsilon_n\\
&= h(Y_1^n) - h(Z_1^n) + 2n\epsilon_n\\
&\le \frac{n}{2}\log(1 + (1 + a_1^n)P/N_1) + 2n\epsilon_n
\end{aligned}$$

where (a) follows from Fano's inequality under the assumption that X_1^n, X_2^n can be recovered at receiver 2 and (b) follows from Fano's inequality under the assumption that $X_1^n + a_2 X_2^n$ can be recovered at receiver 1. In the limit $n \to \infty$ we have the upper bound

$$R_1 + R_2 \le \frac{1}{2}\log\left(1 + \frac{(1+a_1^2)P}{N_1}\right). \tag{9}$$

Using the assumption in the theorem that $(\mathcal{C}_1^{(n)}, \mathcal{C}_2^{(n)})$ are good computation codes for the GMAC$(1, a_1, N_1)$, it means that R_1, R_2 should satisfy

$$R_1 + R_2 = \frac{1}{2}\log(1 + (1+a_1^2)P/N_1 + r) \tag{10}$$

for some $r > 0$ by Definition 3. However, this implies that decoder 2 *cannot* decode both codewords reliably. Otherwise the achievable sum rate in (10) directly contradicts the upper bound in (9). Since this argument holds for any values of a_2, N_2, we conclude that the same sequence of codes cannot be used as multiple-access codes for the GMAC$(1, a_2, N_2)$, regardless of the capacity region of the this channel. ■

The counterpart of Theorem 3 is given as follows.

Theorem 5 (Duality for the general model): Let $(\mathcal{C}_1^{(n)}, \mathcal{C}_2^{(n)})$ be a sequence multiple-access codes for the GMAC$(1, a_2, N_2)$. Then this sequence of codes cannot be good computation codes w. r. t. the sum $X_1^n + a_2 X_2^n$ for the GMAC$(1, a_1, N_1)$, for *any* choice of a_1, a_2, N_1, N_2.

The proof follows with a similar argument as in the proof of Theorem 3.

V. Discussions

The duality results in the previous section reveal some fundamental properties of codes using concise arguments. In this section we give an alternative proof, which could serve as a more intuitive explanation of the results.

We explain why the codes which are good for computation cannot be used as multiple-access codes. In fact, we will give a result for a more general Gaussian MAC model

$$Y^n = f(x_1^n, x_2^n) + Z^n \tag{11}$$

where $f : \mathbb{R}^n \times \mathbb{R}^n \mapsto \mathbb{R}^n$ maps the channel inputs x_1^n, x_2^n to some element in $\mathbb{R}^n$, *elementwise*. As usual, both channel inputs are subject to an average power constraint $\sum_{i=1}^n x_{ki}^2 \le nP$, $k = 1, 2$, and the variance of the white Gaussian noise is assumed to be unit. Choosing $f(x_1^n, x_2^n) = h_1 x_1^n + h_2 x_2^n$ gives the usual Gaussian MAC for some channel gains $h_1, h_2 \in \mathbb{R}$.

Lemma 1 (Lower bound on the error probability): Consider a (generalized) Gaussian MAC channel in (11) where the decoder wishes to decode both codewords X_1^n, X_2^n. Let $P_e^{(n)}$ denote the decoding error probability, then it holds that

$$P_e^{(n)} \ge 1 - \frac{H(f(X_1^n, X_2^n))}{H(X_1^n, X_2^n)} - o(n).$$

Proof: Fano's inequality states

$$H(X_1^n, X_2^n | Y^n) \le H(X_1^n, X_2^n) P_e^{(n)} + 1.$$

On the other hand we have

$$\begin{aligned}
&H(X_1^n, X_2^n | Y^n)\\
&= H(X_1^n, X_2^n) - I(X_1^n, X_2^n; Y^n)\\
&= H(X_1^n, X_2^n) - h(Y^n) + h(Z^n)\\
&= H(X_1^n, X_2^n) - h(Y^n) + h(f(X_1^n, X_2^n) + Z^n | f(X_1^n, X_2^n))\\
&= H(X_1^n, X_2^n) - I(Y^n; f(X_1^n, X_2^n))\\
&\ge H(X_1^n, X_2^n) - H(f(X_1^n, X_2^n)).
\end{aligned}$$

The claim follows by combing the two inequalities. ■

Notice that we always have $H(f(X_1^n, X_2^n)) \le H(X_1^n, X_2^n)$. The above lemma shows that if $H(f(X_1^n, X_2^n))$ is *strictly* smaller than $H(X_1^n, X_2^n)$, then the decoding error probability (of decoding both codewords) is bounded *away* from zero as $n \to \infty$, namely the codes cannot be used as multiple-access codes. This is exactly the reason why the codes which are good for computing $f(X_1^n, X_2^n)$ cannot be used for multiple access for the MAC of the form (11): the codes which are good for computing $f(X_1^n, X_2^n)$ over *any* MAC always satisfy $H(f(X_1^n, X_2^n)) < H(X_1^n, X_2^n)$. This is shown in the following lemma.

Lemma 2: Consider any memoryless multiple access channel given by the conditional probability distribution $W(Y|X_1, X_2)$. If a pair of codes $(\mathcal{C}_1^{(n)}, \mathcal{C}_2^{(n)})$ is good for computing the function $f(X_1^n, X_2^n)$ over this MAC, namely the sum rate is larger than the sum capacity of the MAC, then the following relation holds with strict inequality in the limit $n \to \infty$

$$\frac{1}{n}H(f(X_1^n, X_2^n)) < \frac{1}{n}H(X_1^n, X_2^n)$$

Proof: According to the Definition 3, the sum rate of the codes is larger than the sum capacity of the MAC described by $W(Y|X_1, X_2)$, i.e.

$$R_1 + R_2 > I(Y; X_1, X_2)$$

for any choice of random variables X_1, X_2. It follows that

$$\begin{aligned} H(X_1^n, X_2^n) &= n(R_1 + R_2) \\ &> nI(Y; X_1, X_2) \\ &\overset{(a)}{\geq} I(Y^n; X_1^n, X_2^n) \\ &\overset{(b)}{\geq} I(Y^n; f(X_1^n, X_2^n)) \\ &= H(f(X_1^n, X_2^n)) - H(f(X_1^n, X_2^n)|Y^n) \\ &\overset{(c)}{\geq} H(f(X_1^n, X_2^n)) - n\epsilon_n \end{aligned}$$

where (a) follows due to the memoryless property of the channel. Step (b) uses data-processing inequality and (c) uses Fano's inequality under the assumption that $f(X_1^n, X_2^n)$ can be decoded reliably. We have the claimed result since $\epsilon_n \to 0$ as $n \to \infty$. ■

Lemma 1 and 2 constitute an alternative (even more general) proof of the results in the previous sections. More precisely, we have the following theorem.

Theorem 6: Consider any memoryless multiple access channel given by the conditional probability distribution $W(Y|X_1, X_2)$. If a pair of codes $(\mathcal{C}_1^{(n)}, \mathcal{C}_2^{(n)})$ is good for computing the function $f(X_1^n, X_2^n)$ over this MAC, namely the sum rate is larger than the sum capacity of the channel $W(Y|X_1, X_2)$, then this pair of codes cannot be used as multiple-access codes for the channel $Y^n = f(X_1^n, X_2^n) + Z^n$.

Proof: Lemma 2 shows that if a pair of codes are good for computing the function $f(X_1^n, X_2^n)$ over any MAC, it must hold that $\frac{1}{n}H(f(X_1^n, X_2^n)) < \frac{1}{n}H(X_1^n, X_2^n)$, which is exactly the condition under which the codewords X_1^n, X_2^n cannot be decoded over the MAC of the form (11), as shown in Lemma 1. ■

Notice that the above theorem encompasses the results in Theorem 2 and Theorem 4. The other direction, i.e., a generalization of Theorem 3 and 5, can be established in a similar way.

Theorem 7: Let $(\mathcal{C}_1^{(n)}, \mathcal{C}_2^{(n)})$ be a pair of multiple-access codes for the Gaussian MAC $Y^n = f(X_1^n, X_2^n) + Z^n$, then this pair of codes cannot be good computation codes w. r. t. the function $f(X_1^n, X_2^n)$ over any memoryless multiple-access channel given by the conditional probability distribution $W(Y|X_1, X_2)$.

VI. Acknowledgment

The work of Jingge Zhu was supported by the Swiss National Science Foundation under Project P2ELP2_165137.

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